MODULE THREE

Shape and Space

MODULE THREE – SHAPE AND SPACE

Contents

UNIT 1

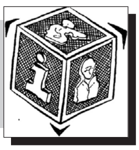
Identifying Shapes

In this unit you will address the following:

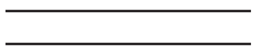
Unit Standard 7464
SO1:
Identify geometric shapes and patterns in cultural products. (shapes of and decorations on cultural products such as drums, pots, mats, buildings, and necklaces.)

To do this you will:
- recognize and name geometric shapes;
- classify shapes according to the nature of their sides and their angles.

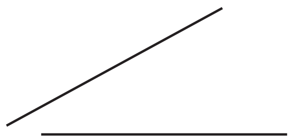
In this unit we explore the definitions and properties of basic geometric shapes, and we will investigate what changes and what stays the same as we enlarge, slide, rotate and flip them.



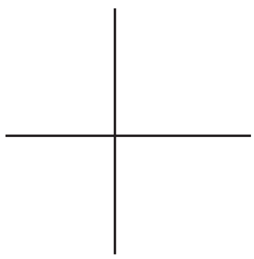
1. Angles



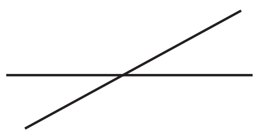
These lines are parallel.



These lines are not parallel.

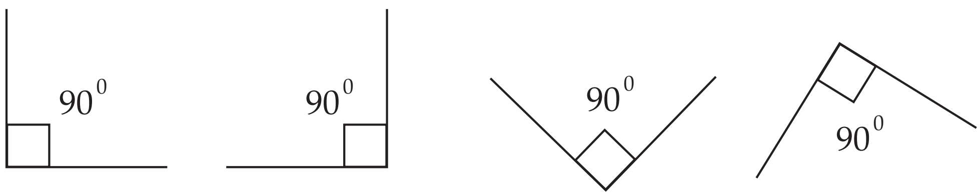


These lines are perpendicular.

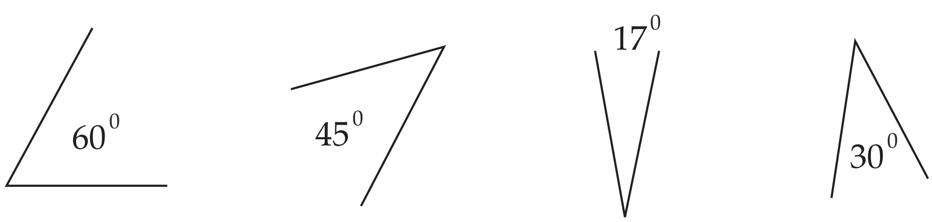


These lines are not perpendicular.

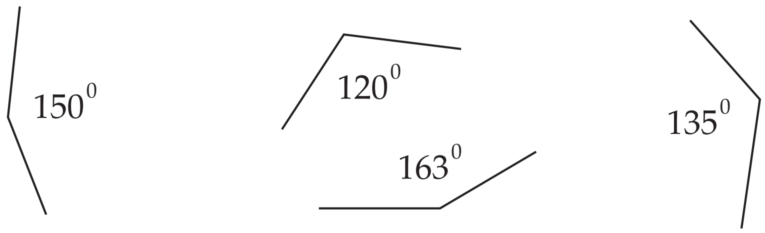
When two lines meet or cross they form an angle. We measure angles in degrees. A circle contains 360 degrees (or 360°). A right angle is $\frac{1}{4}$ of a circle and has 90°.



An acute angle measures less than 90°. The following are all acute angles:



An obtuse angle measures more than 90°. These are all obtuse angles:

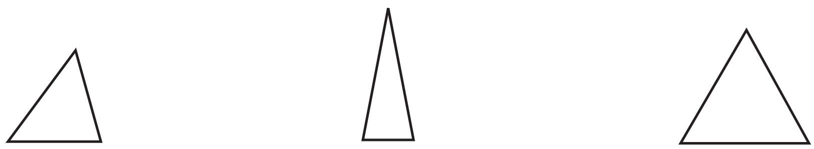


2. Triangles

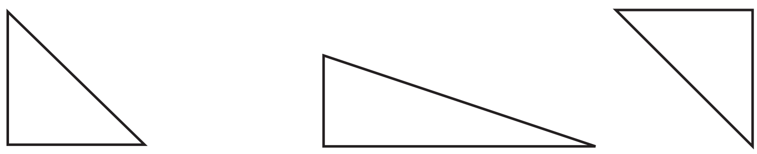
"Triangle" means "three angles." Every triangle has three angles that add up to 180. One way that triangles are classified is by their angles.

Angles of triangles

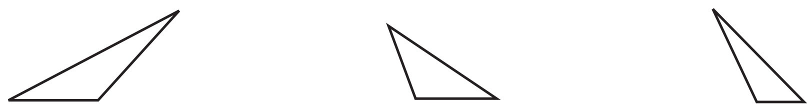
In acute triangles each angle measures less than 90°. This includes equilateral triangles, which have all three angles equal to 60°.



A right triangle has one angle that measures exactly 90°.

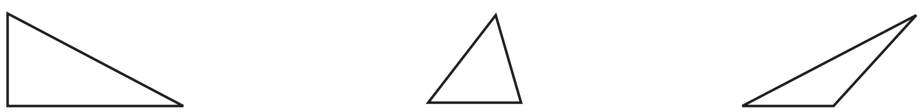


An obtuse triangle has one angle that measures more than 90°.

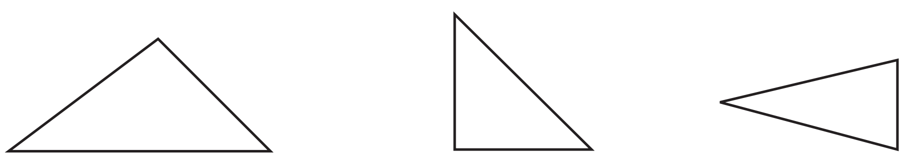


Length of the sides

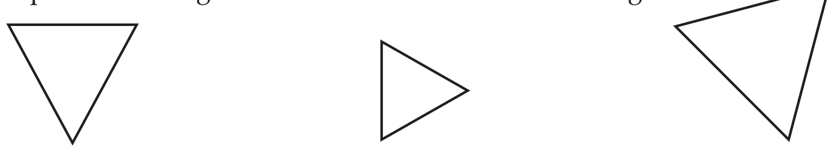
Another way in which triangles are classified is by the lengths of their sides. Scalene triangles have three sides of different lengths.



Isosceles triangles have two sides the same length.



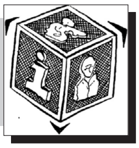
Equilateral triangles have all three sides the same length.



Activity 1:
Triangles

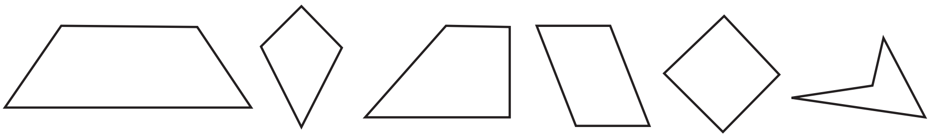
1. Draw two of each kind of triangle described above. Label them and explain why you have given them these names.
2. Compare your drawings with a partner.

Time needed
40 minutes

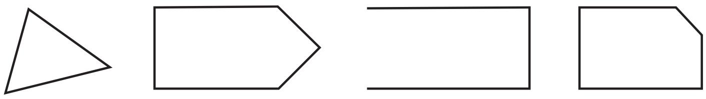


3. Quadrilaterals

All of these shapes are quadrilaterals.



These shapes are not quadrilaterals.



Activity 2:
Quadrilaterals

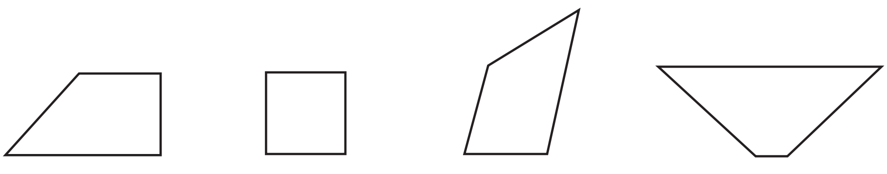
Work with a partner to explore the properties of the polygons below.

1. Discuss the properties of a quadrilateral with your partner.
2. Work together to write a definition.

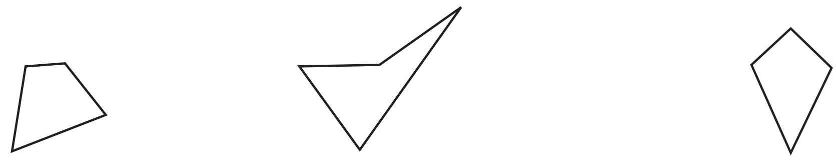
Time needed
70 minutes

Trapezium.

There are many special kinds of quadrilaterals. All of the shapes below are trapeziums.

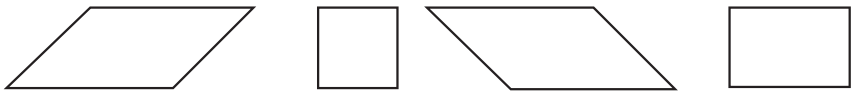


These quadrilaterals are not trapeziums.



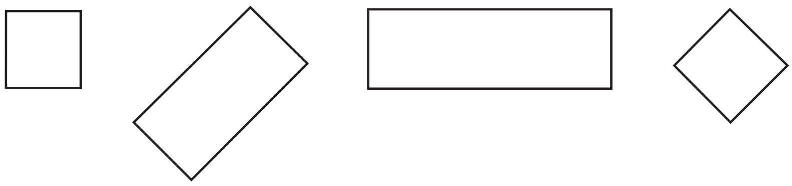
3. Discuss the properties of a trapezium with your partner.
4. Work together to write a definition.

Parallelogram.
All of the quadrilaterals below are parallelograms.



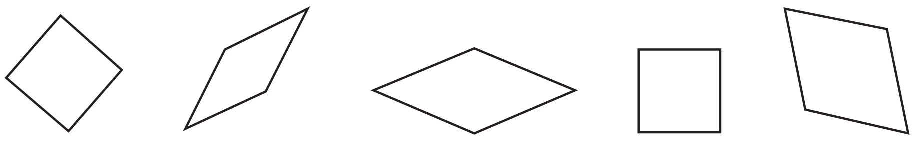
5. Discuss the properties of a parallelogram with your partner.
6. Work together to write a definition.

Rectangles.
All of the quadrilaterals below are rectangles.



7. Discuss the properties of a rectangle with your partner.
8. Work together to write a definition.

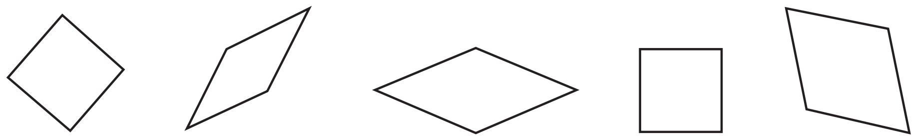
Rhombus.
Each of the shapes below is a rhombus.



9. Discuss the properties of a rhombus with your partner.
10. Work together to write a definition.

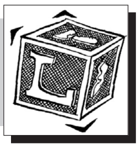
Square.

Each of the shapes below is a square.



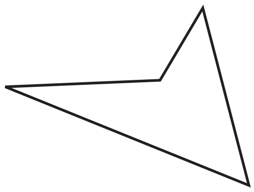
11. You have written definitions for the quadrilateral, trapezium, parallelogram, rectangle, and rhombus. Which of these categories includes squares?

12. If you add up the measurement of the four angles in a square what is the sum? See if you can figure out the sum of the angles for the other types of quadrilaterals.

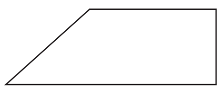


What have you learned?

A quadrilateral is a closed shape with four straight sides.

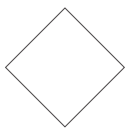


A quadrilateral is a closed shape with four straight sides.

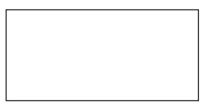


A quadrilateral that has at least one pair of parallel sides is called a trapezium.

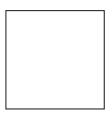
A parallelogram has two pairs of parallel sides, and each pair of opposite sides have equal length.



A parallelogram with four sides the same length is called a rhombus. (A rhombus with four right angles is a square.)



A parallelogram with four right angles is a rectangle.



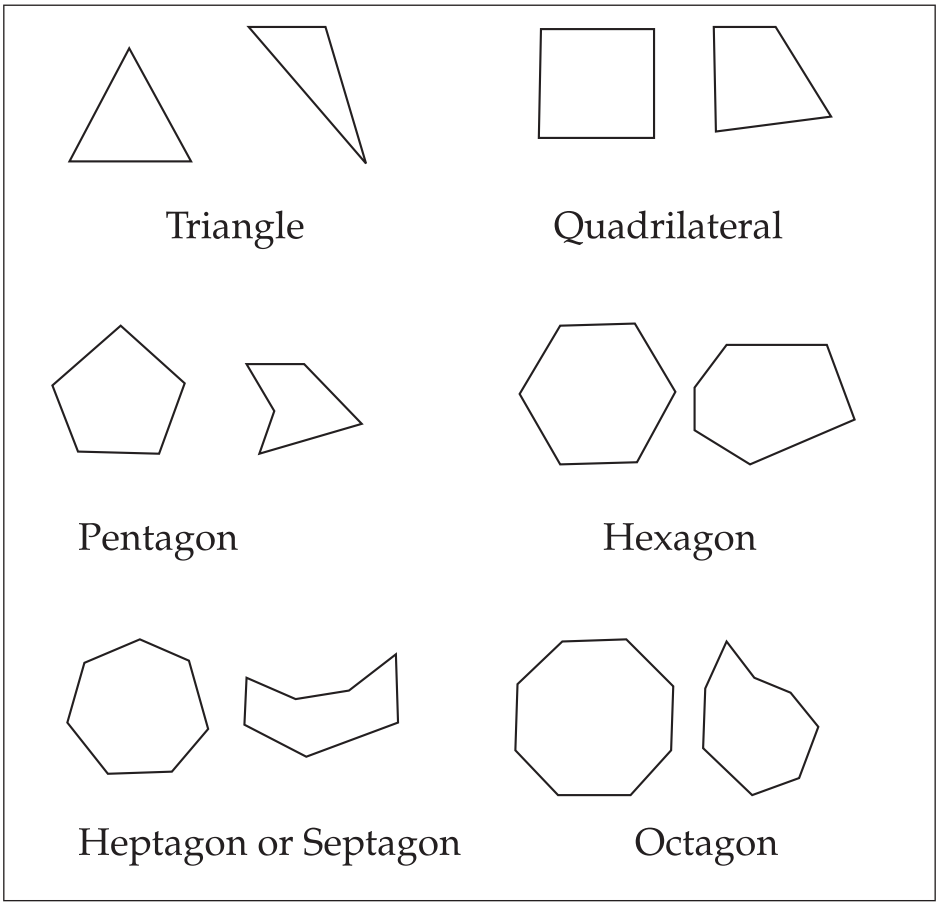
A rectangle with four equal sides is a square. A rhombus with four right angles is a square.

4. Polygons

Triangles and quadrilaterals are two of the many kinds of polygons. A polygon is a plane figure with three or more sides. When all of the sides are the same length and all of the angles are the same, as in an equilateral triangle, or a square, the shape is called a regular polygon. Polygons are named for the number of sides they have.

DICTIONARY:

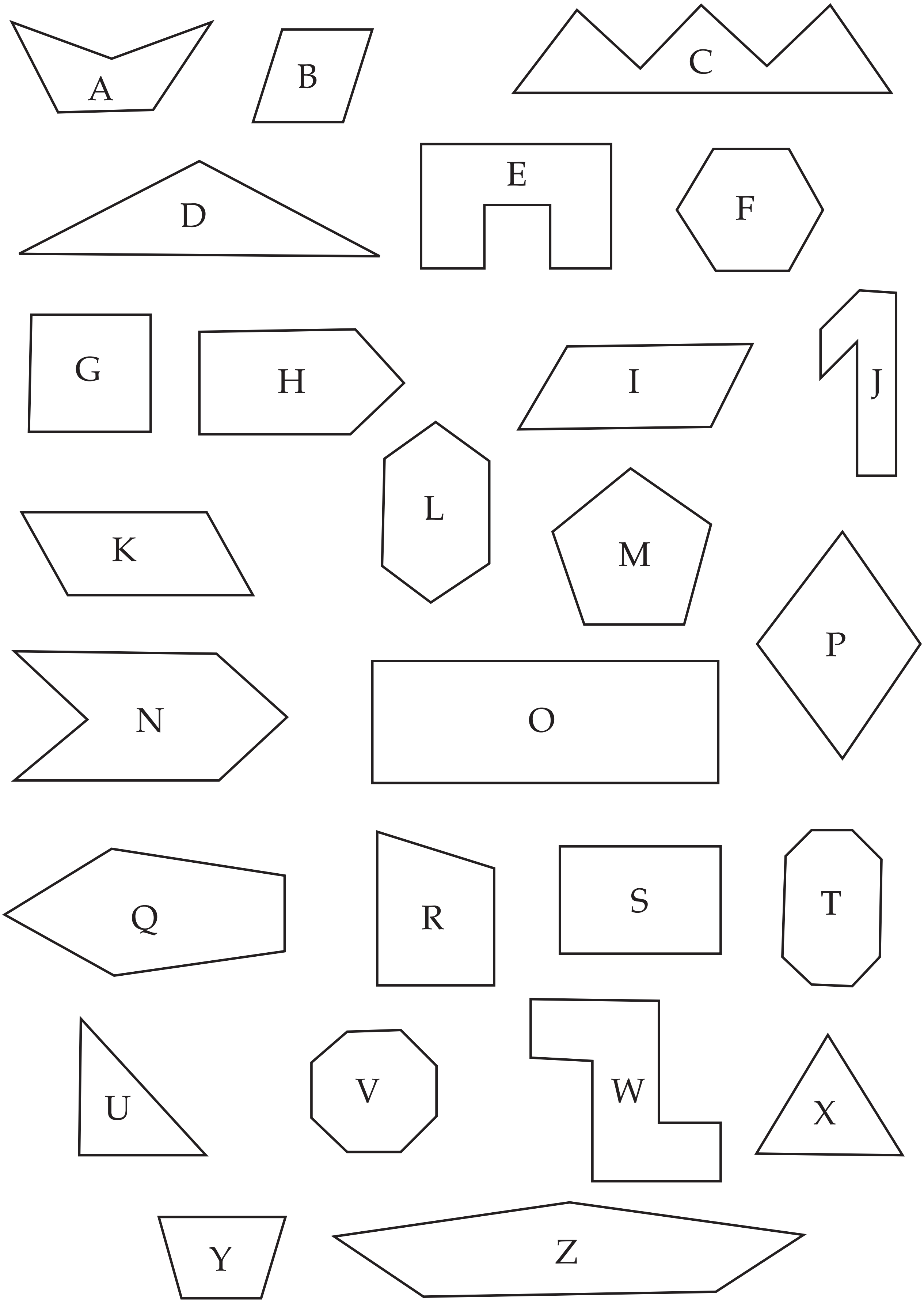
plane – flat



Time needed 40 minutes

Activity 3:
Polygons

1. Look at the Polygon Shape Sheet on the next page. Make a list of the shapes from A-Z. For each shape, write "regular" if all the sides are of equal length and all the angles are equal. Write "irregular" if they are not. Then write the name for the type of polygon, based on the number of sides.
2. Compare your list with your partner. Are there any differences? Can you agree?



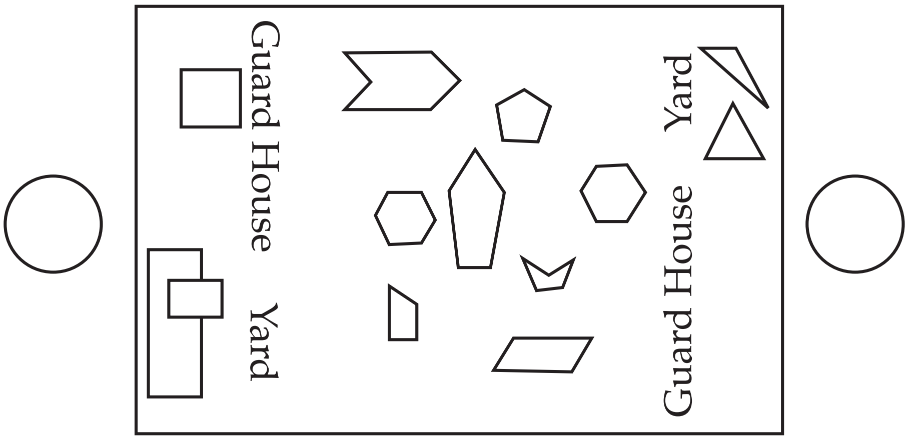
Polygon Shape Sheet

Time needed
60 minutes

Activity 4: Polygon Ambush

You are going to play a game of capturing polygons according to the attributes of their sides and angles.

Preparation:
Cut out the shapes from the Polygon Shape Sheets and place them in the centre of the table. Cut out the property cards and put the SIDES cards in one pile and the ANGLES cards in another. Divide the space in front of you into a yard and a guardhouse, as in the diagram below.



Warm Up:
[Time needed: 10 minutes]

- Pick a card from the SIDES deck. Work with your partner to decide which of the 26 shapes have the property described on the card, and which don't. Try a card from the ANGLES deck.
- One partner picks a card from the SIDES deck. The other partner picks a card from the ANGLES deck. Together sort the shapes into 2 groups: one group with shapes that have both attributes and another with shapes that have only one attribute.

Polygon Ambush Level 1
Take turns.

1. Player 1 chooses one card from either the SIDES deck or the ANGLES deck. She may capture all of the polygons from the centre that have that attribute.
2. After player 1 has finished selecting the shapes she may take, Player 2 may take any remaining pieces with that attribute (that is, any shapes that Player 1 missed).
3. Play passes to Player 2. She chooses one card from either the SIDES deck or the ANGLES deck and then may capture all of the polygons from the centre which have that attribute.
4. Player 1 must now protect any pieces she has already captured which have the

attribute on the card by placing them in the "guardhouse".

5. Player 2 may take any polygons Player 1 has already captured if they match the attribute, and are not in the guardhouse. Player 2 may also take from the guardhouse any pieces that do not have the attribute on the card (that is, any shapes that Player 1 has put in the guard house in error).
6. If Player 2 selects an incorrect piece from the centre, then Player 1 may challenge and take that piece.
7. When there are no more pieces that match the attributes on the card it's the next players turn.
8. Play continues until there are no more pieces. The player with the most pieces wins.

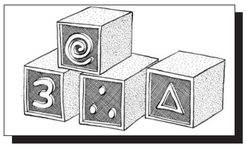
Property Cards	
SIDES	**ANGLES**
All sides are congruent (have the same length).	All angles are congruent (equal).
One and only one pair of opposite sides is parallel.	One and only one angle is a right angle.
All pairs of opposite sides are parallel.	Two or more angles are acute.
There are four sides.	Two or more angles are obtuse.
There are three sides.	One and only one angle is obtuse.
There are more than four sides.	Two or more angles are right angles.
All sides have different lengths.	No angle is a right angle.
Two or more sides, but not all, are congruent (have the same lengths).	If you flip it over you get the same shape.
All pairs of opposite sides are congruent.	All pairs of opposite angles are congruent.
Exactly two sides are equal.	All angles are different.

Polygon Ambush Level 2

The rules are the same as above, except that on each play, the player chooses one card from the SIDES deck AND one card from the ANGLES deck. The players may capture any pieces that have BOTH attributes.

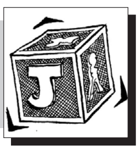
Freestyle Polygon Ambush

Another way to use the polygon shapes to explore attributes is for one player to think of an attribute for the SIDES and an attribute for the ANGLES. Divide the shapes into those that match your attributes and those that don't. The challenge for the other player is to try to guess what attributes you've chosen.



Linking your learning with your ECD work

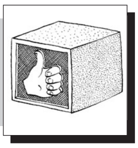
- Choose some of the shapes you have worked with in this unit.
- Make large copies of them on cardboard. Let your learners describe them by the number of sides and the number of corners they each have.
- Encourage them to look for similarities and differences e.g. shape with 4 sides and 4 corners, shapes with 3 sides and 3 corners.
- They may know the names of some of the basic shapes like squares and triangles. The focus of the activity should rather be for children to describe and compare the different characteristics of the shapes, rather than giving them the correct mathematical names. Too often we encourage children to learn and chant the names of these shapes in parrot fashion. This is meaningless if they do not look closely at their unique characteristics and think about what makes them the same and different.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about geometric shapes?
b. How do you think you will be able to improve your understanding of classifying shapes?
c. Write down one or two questions that you still have about classifying shapes.
d. How will you use what you learned about shapes in your every day life and work?

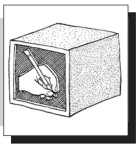


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
√1. Recognize and name geometric shapes.				
2. Classify shapes according to the nature of their sides and their angles.				



Assignment 1: Is there a heptagon in the house?

Cut two pieces of A4 paper in quarters, so you'll have 8 pieces. On two pieces write "triangle", and on two write "quadrilateral". On each of the 4 remaining pieces write "pentagon", "hexagon", "heptagon", and "octagon". Look around your house, your neighborhood, and your workplace for examples of each shape. Make a sketch and write a few words to explain where you see the shape. It can be a regular or an irregular polygon. Quadrilaterals and triangles are everywhere and easy to spot. Finding the other polygons is more challenging, and more fun. Look at buildings, decorations, beadwork, weaving, plants, boxes, signs, machines, and shadows – everything around you. Look at large things and small things. When you have done this, share your sketches and your examples with a partner.

UNIT TWO

Tangrams

In this unit you will address the following:

Unit Standard 7464
SO1:
Identify geometric shapes and patterns in cultural products. (shapes of and decorations on cultural products such as drums, pots, mats, buildings, and necklaces.)

SO2:
Analyze similarities & differences in shapes & patterns, & effect of colour, used by cultures. (analyze similarities and differences in shapes and patterns, and the effect of colour, used by different cultures.)

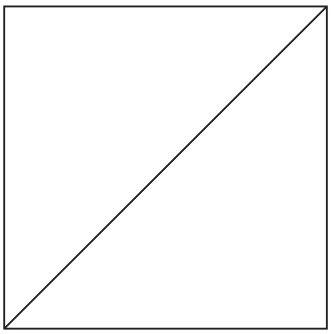
To do this you will:
- follow instructions to cut out a 7-piece Tangram puzzle;
- identify which of the shapes are similar and which are congruent;
- use the shapes in different transformations and combinations to build new shapes and patterns of your own and by following the outlines of different shapes given to you.

Time needed
90 minutes

Activity 1:
Tangram instructions

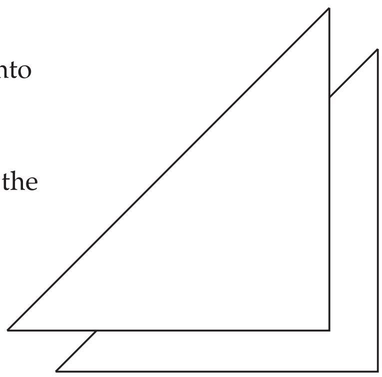
Work alone

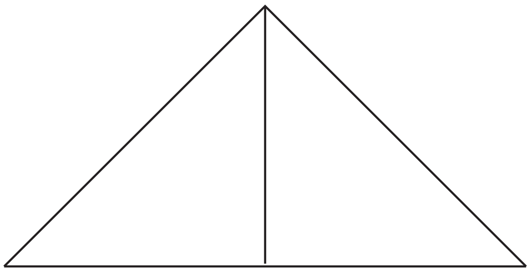
1. Follow these instructions step-by-step.



Take a square of paper, fold carefully along the diagonal and cut along the fold to get two triangles.

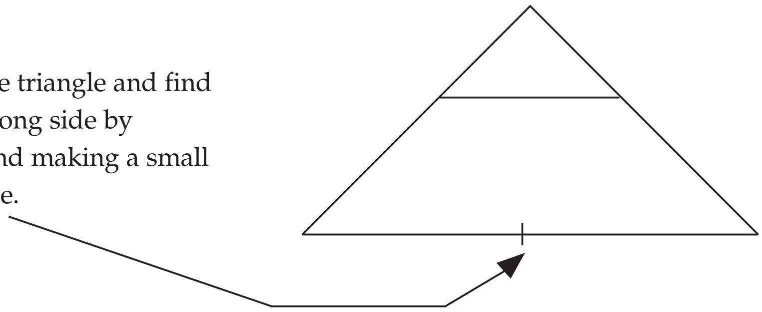
If you rotate one of the triangles and slide it onto the other triangle you will see that they are exactly the same size and shape. In mathematical terms, shapes which are exactly the same are called congruent.

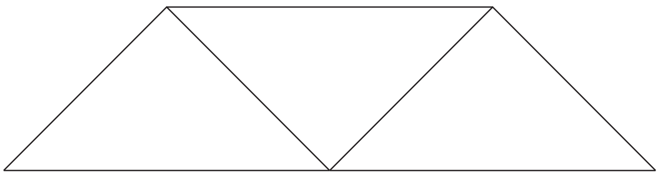




Put one triangle aside and fold the other carefully in half, then cut along the fold.

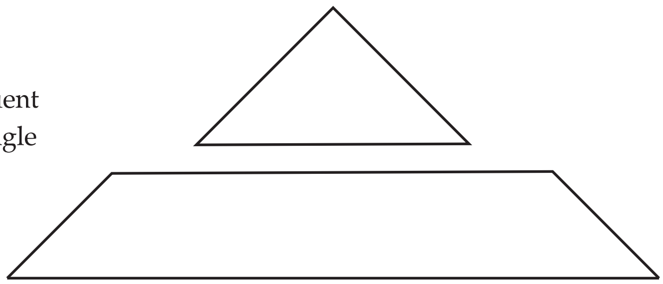
Now, take the large triangle and find the middle of the long side by folding it in half and making a small pinch in the middle.

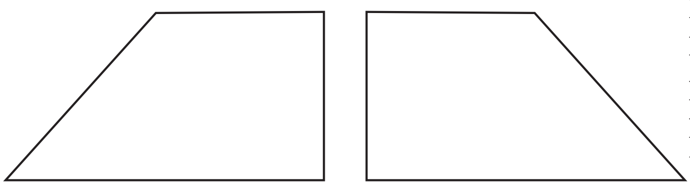




Then fold the right angle down to the middle of the base. Cut along the crease.

In this new triangle congruent or similar to the other triangle you have?



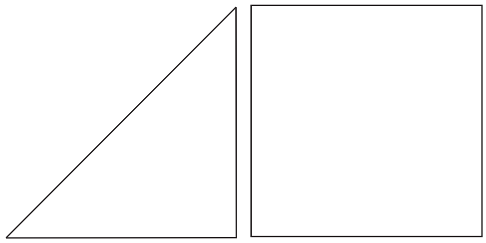


Fold the trapezium in half and cut it into two pieces. Note that the new pieces are also trapeziums.

Are these two pieces congruent? If so, one piece should exactly cover the other. To prove that the triangles were congruent you needed to change, or transform the shape in two ways. You rotated the shape, and slid it on top of the other piece. Sliding in mathematical terms, is called translation.

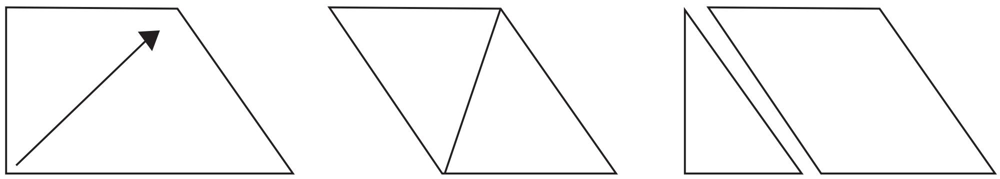
Can you make the trapeziums fit using only these two transformations? Try it.
Do you need another transformation? Try flipping one of the pieces over. Flipping a shape over is called reflection in mathematics.

Now, take one piece and fold it at the middle of the long side, then cut along the crease to make a square and another triangle.

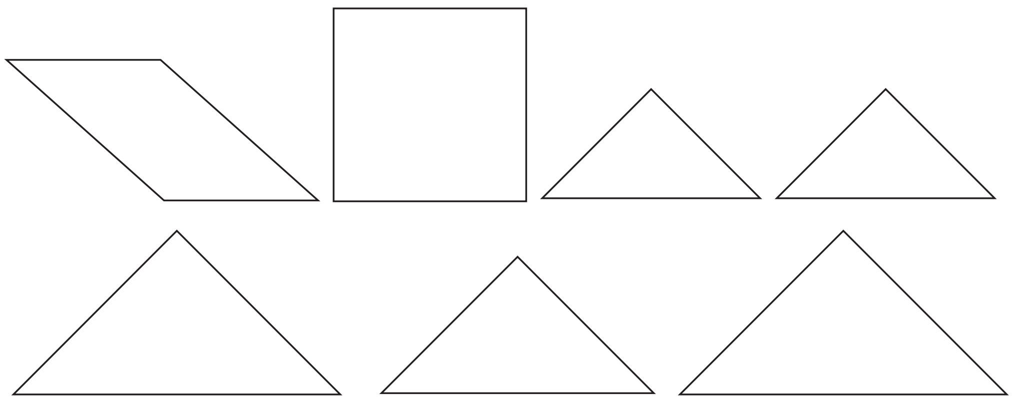


And here's the last cut. Fold the right angle on the long side up to meet the obtuse (wide) angle. Fold and cut to make a parallelogram and another triangle.

Fold Crease Cut



Now you have 7 shapes. Which ones are congruent? Which are similar? Which are regular polygons? Which are the same when you flip them over, and which are different?

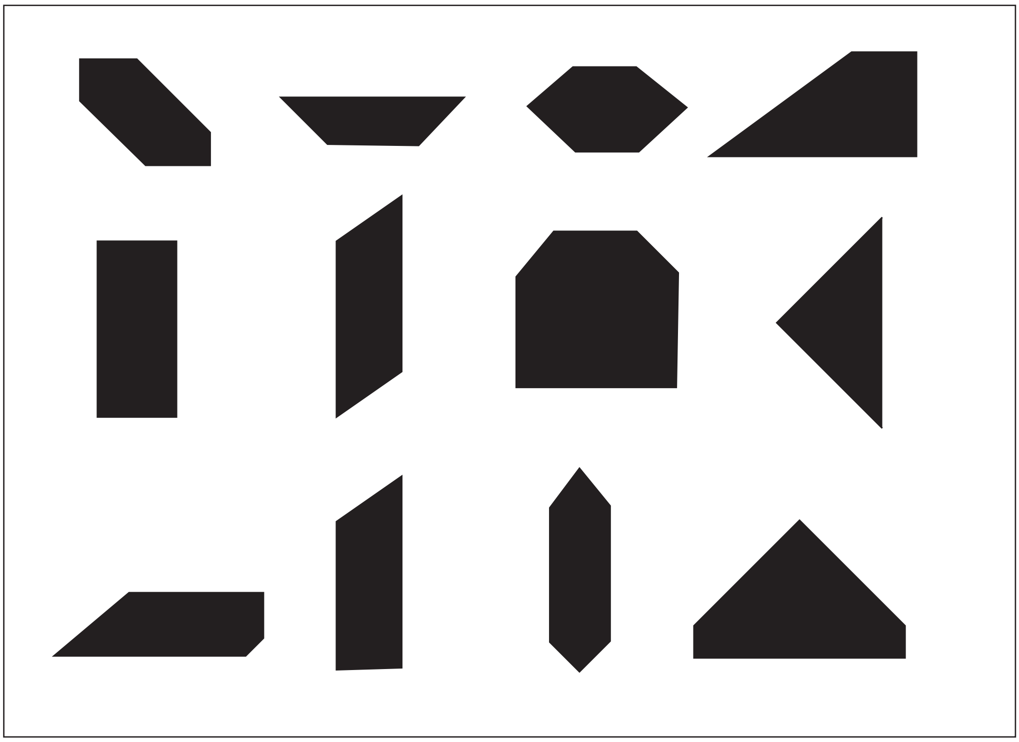


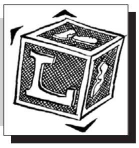
These pieces are the seven "tans" of the famous Chinese puzzle called "tangram". Exploring tangram puzzles is a great way to build spacial visualization skills.

Can you put these pieces back together to form the square you started with? If you need help, look back at thee process you used to cut up the square into pieces.

Next, make a puzzle for your partner to solve. Put the pieces together in any way you like, then carefully trace the outline. Give it to your partner to try to solve while you are trying to solve the one your partner has invented.

You can make your puzzle as complicated as you like, but often the simpler shapes are the most challenging. Here are a few of the shapes you can make using all 7 pieces. Try some of them at home. Your family and friends might enjoy them too.



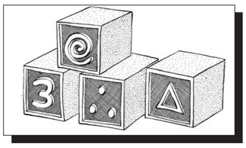


What have you learned?

You have managed to cut out a seven piece tangram puzzle by following the steps. You could probably name each shape using the correct mathematical name and you discovered which shapes are similar and which are congruent.

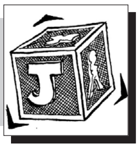
Now you can change the shapes in different ways to build new shapes. You realized that you can describe these changes using mathematical names, e.g. rotations and reflections.

By working with the shapes you became familiar with (used to) the properties of the different shapes. You could see the lengths of their sides and angles and you managed to build new shapes and patterns, following the outlines of shapes given.



Linking your learning with your ECD work

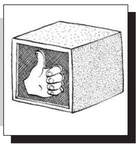
- Young learners will not be able to work in the same way that you did with tangram pieces.
- But you can give them cardboard pieces to explore ways of making and building patterns on their own.
- Follow the same steps to make your shapes and then stick them onto cardboard so that they will last longer.
- Let then make pictures or just build shapes on their own such as building a square from two triangles.
- Let the children explore the shape pieces freely on their own. Encourage them to talk about and explain to others what they find out while exploring.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about tangrams?
b. How do you think you will be able to improve your understanding of tangrams?
c. Write down one or two questions that you still have working with tangrams.
d. How will you use what you learned about tangrams in your every day life and work?

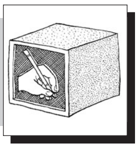


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Follow instructions to cut out a 7-piece Tangram puzzle				
2. Identify which of the shapes are similar and which are congruent				
3. Use the shapes in different transformations and combinations to build new shapes and patterns of my own and by following the outlines of different shapes given to me.				



Assignment 2:

Make as many shapes out of the tangram as you can. Draw the ones you make.

UNIT THREE

Symmetry and Transformations

DICTIONARY:

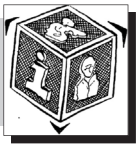
symmetry – balance

In this unit you will address the following:

Unit Standard 7464
SO1:
Identify geometric shapes and patterns in cultural products. (shapes of and decorations on cultural products such as drums, pots, mats, buildings, and necklaces.)

SO2:
Analyze similarities & differences in shapes & patterns, & effect of colour, used by cultures. (analyze similarities and differences in shapes and patterns, and the effect of colour, used by different cultures.)

To do this you will:
- identify shapes that have symmetry along one or more lines;
- identify shapes that have rotational symmetry;
- identify shapes that have translational symmetry;
- describe and explain transformations: reflection, rotation and translation are the three ways we can move objects in the plane without changing their shape or size.



1. Symmetry

We have seen that we can classify figures in many ways:
- by the number of sides they have
- by the properties of their angles.

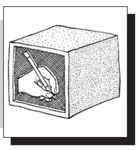
Now let's explore another quality: symmetry.

Start with this exercise.

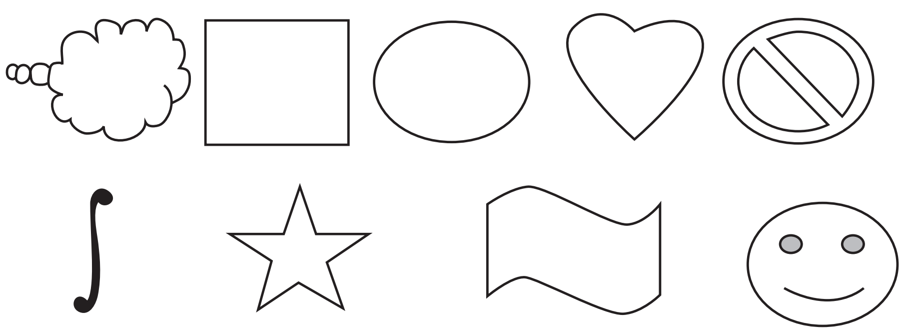
Time needed
20 minutes

Activity 1:
Symmetry Brainstorm

Do Activity 1 on a separate paper and add it to your portfolio.



Work with a partner.
Look at the following pictures.



1. Discuss these questions together.
   a. Which shapes have symmetry?
   b. What makes them symmetrical?

What have you learned?

You needed to be creative in that activity. Maybe you found that there are a number of different ways to find symmetry in some of the pictures, and only one way in others. For example if you draw a line down through the heart shape it is balanced on both sides. If you draw a line across the heart shape it is not balanced at the top and bottom. The circle is symmetrical whichever way you cut it but it has to be cut in half.

Activity 2:
Mirror Symmetry

Time needed
20 minutes

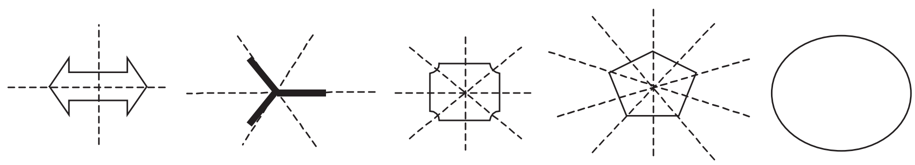
Work with a partner.

For this activity you will need a small mirror.

1. Draw a shape on your paper and draw a line through it.
2. Label the line L.
3. Place the mirror along the line, perpendicular to the paper. (If the paper is on the table the mirror will be standing up vertically.)
4. Discuss what you see in the mirror.

What have you learned?

Part of the shape is in front of the mirror and part will be hidden by the mirror. Perhaps the image that you saw in the mirror looked like the part of the shape on the paper. If so then the shape has mirror symmetry about the line L. Here are some examples.

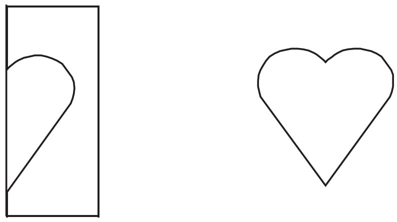


2 lines 3 lines 4 lines 5 lines infinite lines

Maybe you already realized that some shapes have symmetry along more than one line. Explore the mirror symmetry of the shapes in Activity 1.

Paper folding symmetry

There is another way to think about mirror symmetry. Did you ever cut out a heart by folding a paper in half and cutting the outline of half a heart?



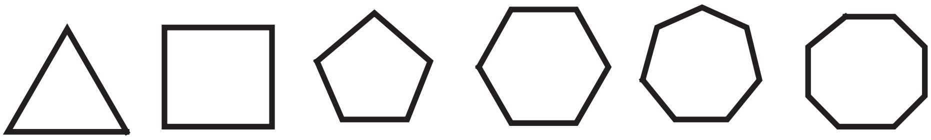
When something has mirror symmetry about a line it also means that if you have a picture of it, and you fold the paper along that line, the part on top will exactly cover the part underneath.

Activity 3:
Paper Folding Symmetry

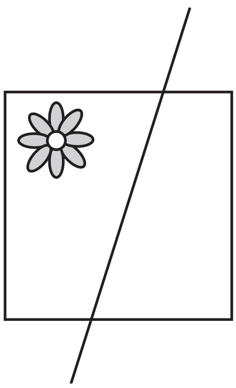
Time needed
35 minutes

Work alone

Trace a shape onto a piece of see-through paper. For each shape, explore how many different ways can you fold the paper so that the half above exactly covers the half below.



Now take a piece of transparent paper and make a small sketch on it. Fold the paper along a line that does not pass through your sketch.



Then trace the figure exactly as it shows through the transparent paper. Unfold and admire your artwork.

Place the mirror along the crease. Does the image look like the figure you drew?

What have you learned?

When you move an object by flipping it – folding it over a line as you have just done – it is called reflection. It gets that name because the new image is like looking at the reflection of the old image in a mirror. As we discussed in the last unit, reflection is one type of transformation. Transformations are the ways we move objects about in a plane.

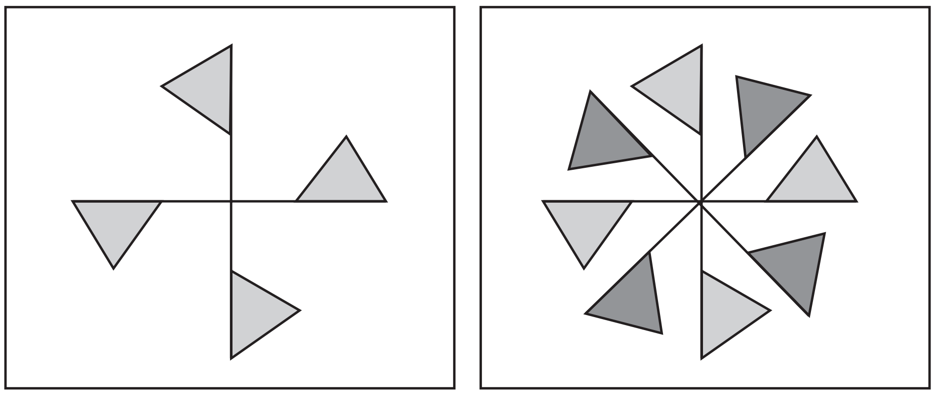
Activity 4: Rotational Symmetry

Time needed 25 minutes

Work alone

When you brainstormed about symmetry in Activity 1, you probably found that some shapes that you thought were symmetrical did not have mirror symmetry. Let's look at another kind of transformation.

1. Sketch the image below on one piece of transparent paper. Then cover it with a second piece and sketch it again.

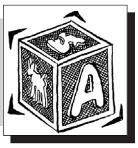


2. Now, put a pin through the center of the two images. Put something underneath, such as a piece of cardboard, to stop the pin from sticking into the table.
3. Turn the top sheet slowly, counterclockwise, around the pin. Look to see whether the figure on top exactly covers the figure on the bottom.

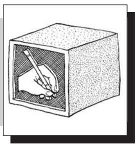
What have you learned?

If you turn the image it means that the pinwheel has rotational symmetry. This movement of turning the figure around a fixed point is called rotation. This is another kind of transformation, another way to move something around in the plane.

To explain how you have rotated something you have to talk about the centre of rotation (where you put the pin) and the angle of rotation (how far you turned it). Mathematicians call counter clockwise rotation positive and clockwise rotation negative. So a rotation of 90° means 90° counterclockwise. A rotation of –90° means turning 90° clockwise.



Time needed
20 minutes

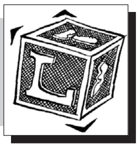


Activity 5:
Mirror and rotational symmetry

Do this activity on a separate paper and add it to your portfolio

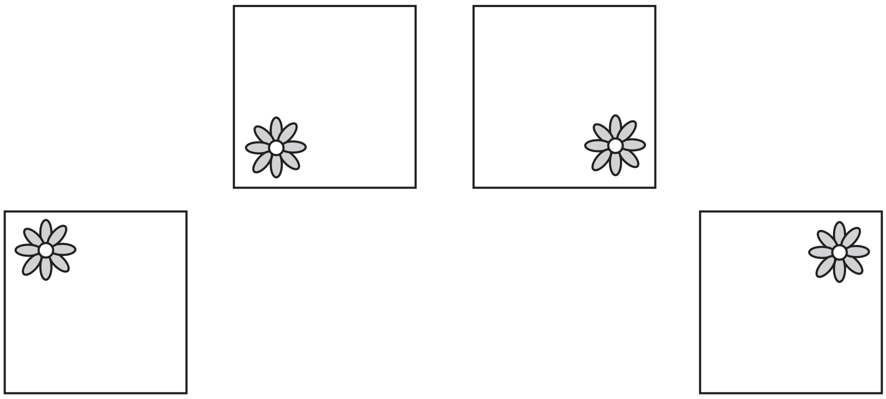
Work alone

1. Think about which of the figures from Activity 1 both mirror symmetry and rotational symmetry have.
2. Which have only rotational symmetry?
3. Where is the center of rotation?
4. Use your tracing paper to check your answer.

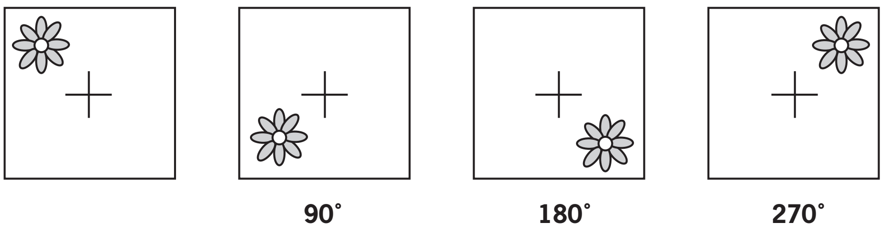


What have you learned?

The centre of rotation is not always at the centre of the object. If we draw a flower near one corner of the paper, put the pin through the centre of the flower, and rotate the paper, taking a snapshot every 90^0, the motion would look like this:



Or, with the same sketch we could put the pin at the center of the paper, but it wouldn't be in the center of the flower. That rotation would look like this:



The next quarter turn would, of course, bring it back – full circle – to where it started.

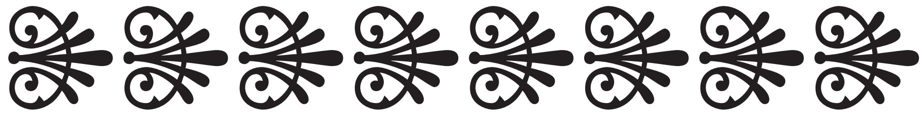
Time needed
30 minutes

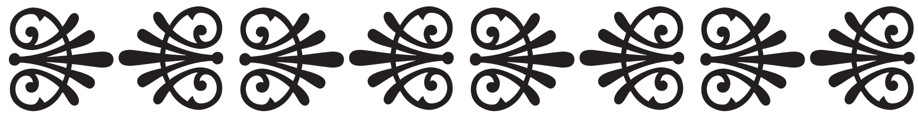
Activity 6:
Translational Symmetry

Work alone

Think back to Activity 1.

1. If we included patterns like the two below would you have said they have symmetry?





2. Study the first design. Make a copy of a portion of it on your tracing paper. Now slide the paper along and observe how the copy is aligned with the original each time you move by the space of one symbol.
3. Now study the second pattern. Not all of the elements are the same. What is the pattern that repeats itself in this line? Draw the pattern on your tracing paper and observe what happens as you slide along.

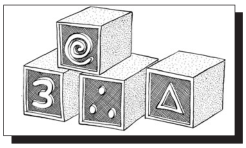
What have you learned?

The movement of sliding the image without flipping or turning is called translation. In this case you are moving horizontally, but the translation could be horizontal, vertical or any combination of the two. To describe a translation you have to talk about the direction you are sliding in and how far you are going.

Reflection, rotation and translation are the three ways you can move in the plane without changing the size or shape of the image. Mathematicians call them "isometric" transformations because iso means "the same" and "metric" means measure.

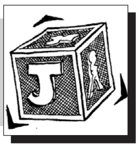
By working through the activities in this unit you should now be able to:

- Identify shapes that have symmetry along one or more lines (mirror symmetry or reflection symmetry)
- Identify shapes that have rotational symmetry
- Identify shapes that have translational symmetry
- Identify shapes or patterns that have more than one kind of symmetry
- Explain the meaning of transformations: reflection, rotation and translation as the three ways we can move objects in the plane without changing their shape or size.



Linking your learning with your ECD work

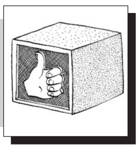
- Design a poster for your classroom walls using natural objects like butterflies, which have symmetry. Let the children talk about which parts are the same, where the pattern is repeated etc.
- Children collect and talk about natural objects or patterns in cloth that have symmetry.
- Make butterfly pictures. Children paint on only one side of a paper. They fold the paper down the middle and make it flat. Open the paper to see a symmetrical pattern.
- Older children can fold a piece of paper and cut a design along the fold line. They will see a symmetrical paper cut-out when they open up their paper.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about symmetry?
b. How do you think you will be able to improve your understanding of symmetry?
c. Write down one or two questions that you still have about symmetry.
d. Write down one or two questions that you still have about transformations.
e. How will you use what you learned about symmetry in your every day life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Identify shapes that have symmetry along one or more lines.				
2. Identify shapes that have rotational symmetry.				
3. Identify shapes that have translational symmetry.				
4. Describe and explain transformations: reflection, rotation. and translation are the three ways we can move objects in the plane without changing their shape or size.				

UNIT FOUR

Polyhedra

DICTIONARY:

constructing – making
3-dimensional – shape with length, width and depth

In this unit you will address the following:

Unit Standard 7464
SO1:
Identify geometric shapes and patterns in cultural products. (shapes of and decorations on cultural products such as drums, pots, mats, buildings, and necklaces.)

To do this you will:
- recognize, name, build, draw, and identify the properties of many polyhedra, including:
  - o Platonic Solids
  - o Prisms
  - o Pyramids
  - o Irregular Polyhedra.

1. What is a polyhedron?

A closed flat figure with 3 or more straight sides is called a polygon. A solid shape with 4 or more flat, faces is called a polyhedron. Each face is a polygon.

Activity 1:
Constructing Polyhedra from Polygons

Time needed
90 minutes

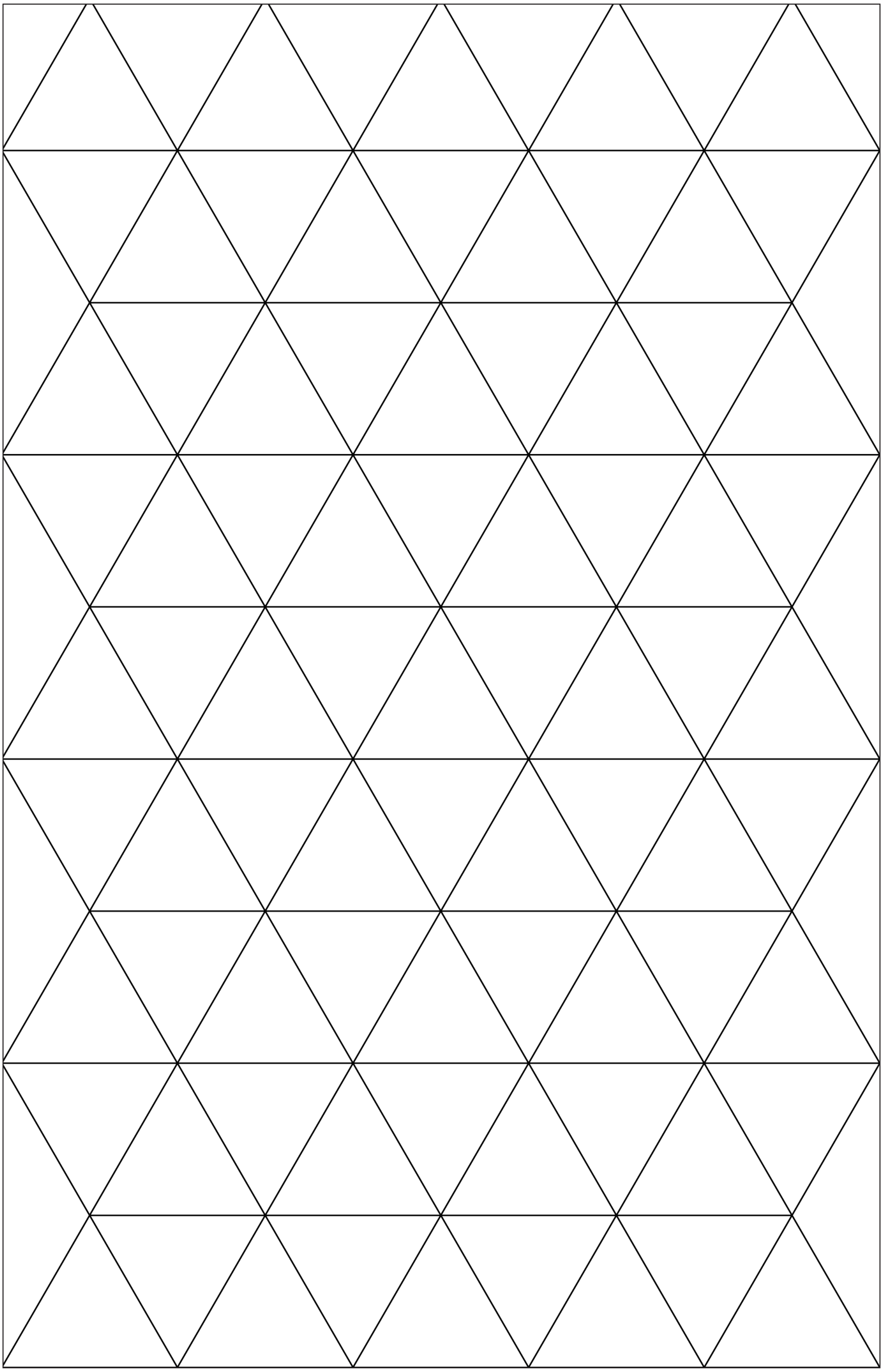
What you need:
- Copies of patterns for squares, triangles, pentagons and hexagons (on page 31, 32, 33 and 34)
- Scissors
- Cellotape

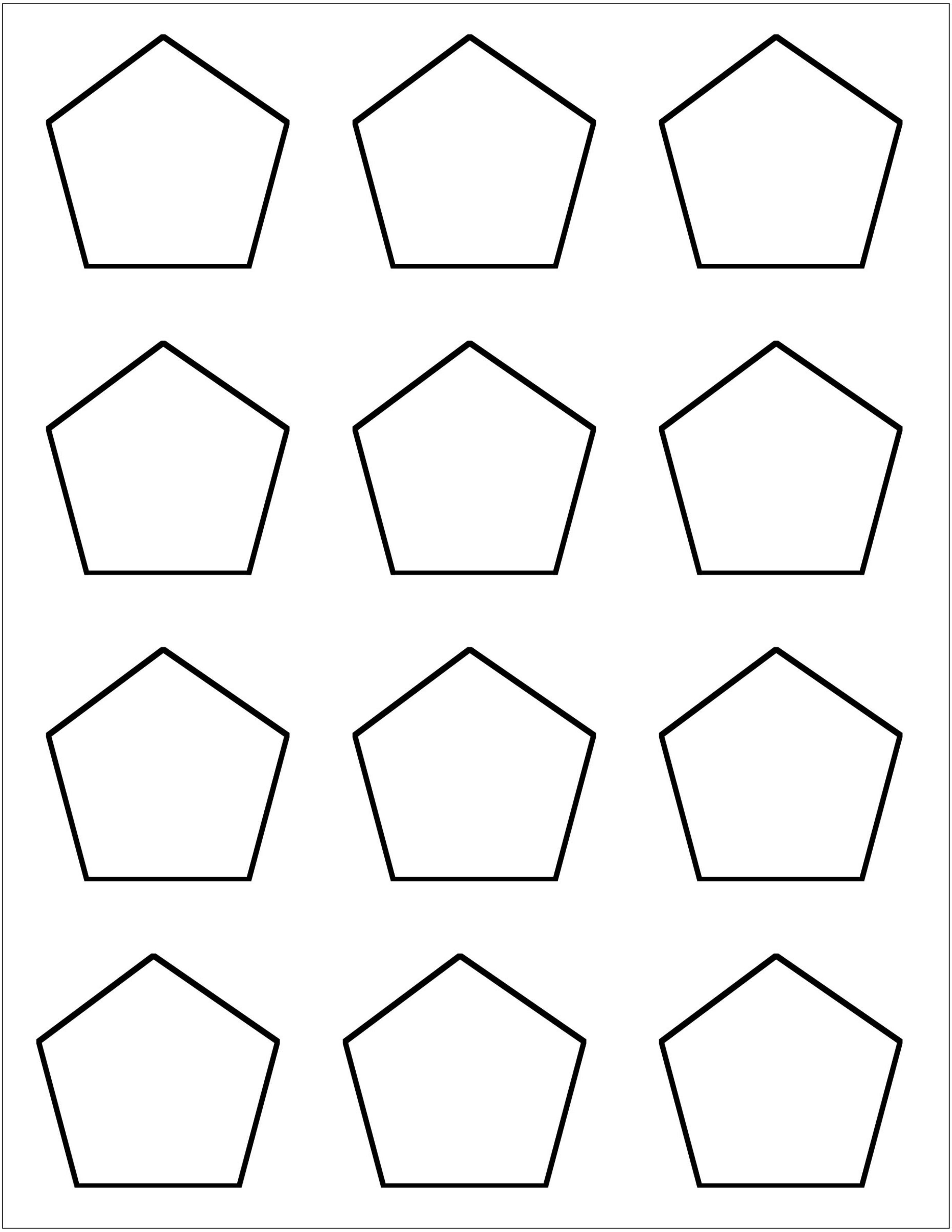
Work with a Partner
1. Cube. How many faces does a cube have? What shape are they? Cut out the appropriate number of squares. Using the squares and cellotape, build a cube. Is there any other solid shape that you can make using only squares for all of the faces (with no gaps)?

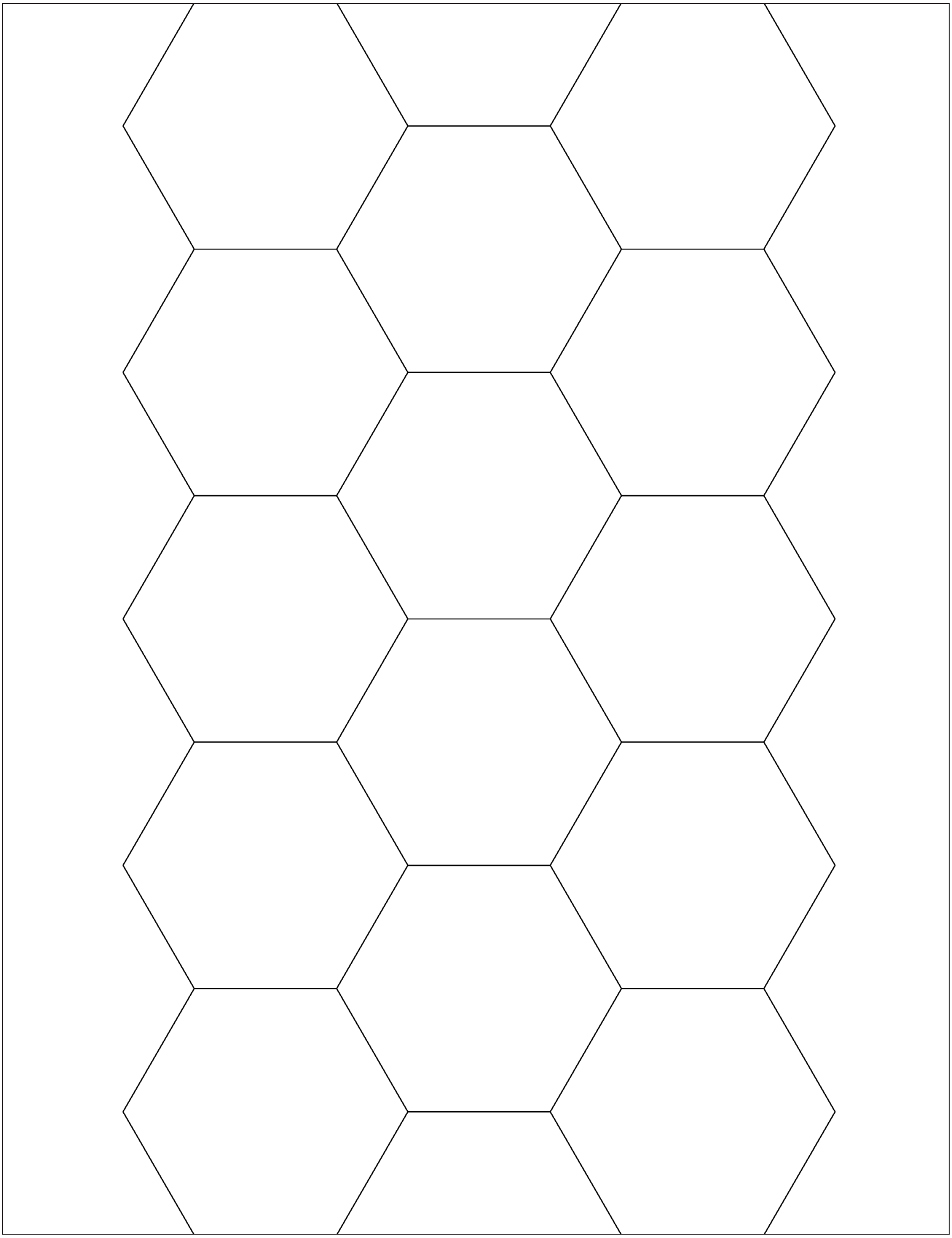
2. Shapes with triangular faces. Now cut out some equilateral (all sides the same length) triangles. Using the triangles and cellotape, how many shapes can you build that have only equilateral triangles for faces?

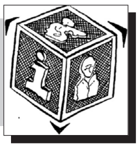
3. Shapes with pentagonal faces. Cut out some pentagons. Once again explore what 3-dimensional (3-D) shape(s) you can make with all the sides as pentagons.

4. Cut out some hexagons. Can you make any 3-D shapes with only hexagons as faces?





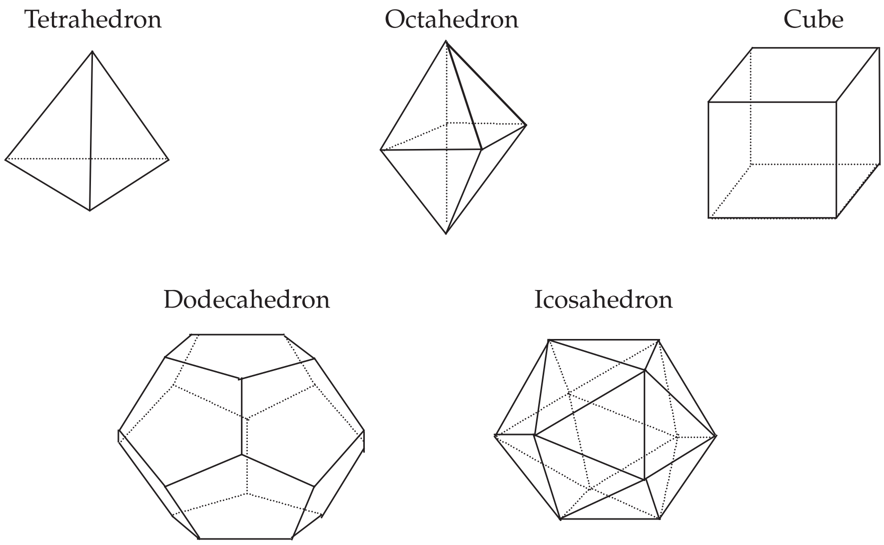




2. Platonic Solids

It is possible to make a polyhedron with any number of faces, but there are only five shapes that have all their faces the same shape and all their angles equal.

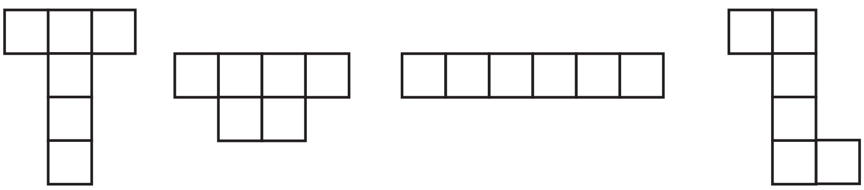
We call these five shapes regular polyhedra, or the platonic solids.

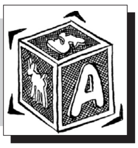


Think about a cereal box. How did it become a box? It started as a flat sheet of cardboard that was folded up to make the box. Try to imagine what the cardboard will look like if you unglue the box and lay it flat. This flat cardboard is the pattern for the cereal box. This kind of pattern – the flat shape that can be folded up to make the 3 dimensional shape – is called a net.

Think about the way you put the six squares together to make a cube. How many different ways can you unfold the cube to make a net?

Look at the patterns below. Which ones can be folded to make a cube? Which ones cannot? There are 11 different ways to make a net for a cube. How many can you find?





Time needed
90 minutes

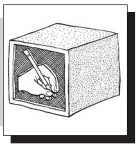
Activity 2:
Constructing Platonic Solids from Nets

What you need:
- Nets of shapes (at the end of this unit)
- Scissors
- Cardboard
- Glue

Work alone
You are now going to build these shapes in two different ways. First, we'll make models out of paper, then out of tooth picks and prestik or sweets like jelly tots.

At the end of this unit, you will find a picture of 5 different nets. You can copy and cut these out to build each of these platonic solids.

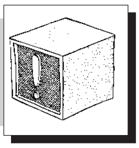
1. First stick each page onto a stiffer piece of cardboard.
2. Cut the nets out neatly along the cutting line.
3. Fold each edge along the fold line and paste the edges neatly together with glue stick.



When you have finished making your platonic solids write task 4. on a separate paper and add it to your portfolio.

4. When you are finished discuss each one and work though these tasks:
   a. Describe each shape by the number of faces each one has and the name of the face (polygon) used to build each shape.
   b. Describe the properties of the different faces: the number of sides and angles they have.
   c. Write clear mathematical definitions to explain each of the 5 platonic solids.
   d. Record them in a table like this:

Name of platonic Solid	Number of Faces	Number of Vertices	Number of Edges
1 Tetrahedron			

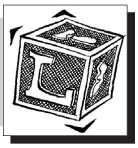


DICTIONARY:

point – apex

Stop and Think

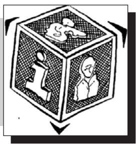
We have said that you can only build five solids (polyhedra) that have all faces the same and all angles the same. But there are many other shapes with different faces and angles that you can use to build polyhedra. Think of a soccer ball for example. What basic 2-D shapes are used to make the rounded shape of a soccer ball? Find a real soccer ball to check if you were right.



What have you learned?

The word polyhedron is used to describe a closed solid figure with 4 or more flat straight sides. Although there are many different kinds of polyhedra, there are only 5 shapes, which we call platonic solids, that are polyhedra with all faces and all angles the same. They are a cube, a tetrahedron, an octahedron, a dodecahedron, and an icosahedron.

You have also learned that a net is a flat design that we can measure, draw, cut and fold to make 3-D models of polyhedra, like the platonic solids.



3. Pyramids

Another special kind of polyhedron is a pyramid. A tetrahedron is a pyramid with a triangular base, but it is also possible to make a pyramid with a square base. A pyramid is a 3-D shape. It has a polygon as a base. The other faces are triangles which meet at a point. A pyramid can have any shaped polygon as its base.

Time needed
20 minutes

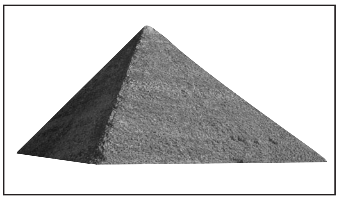
Activity 3:
Pyramid Power

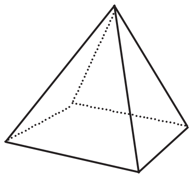
What you need:

- Paper
- Ruler
- Pencil
- Erasor
- scissors
- Glue

The ancient Egyptians built pyramids with square bases.

Work alone





a. Design a net for making a pyramid of any size that has a square base and four congruent triangular sides.
b. When you are satisfied with your net, build the pyramid.

Work in pairs

c. Discuss and compare your pyramid with your partner's. How are your models the same? How are they different?

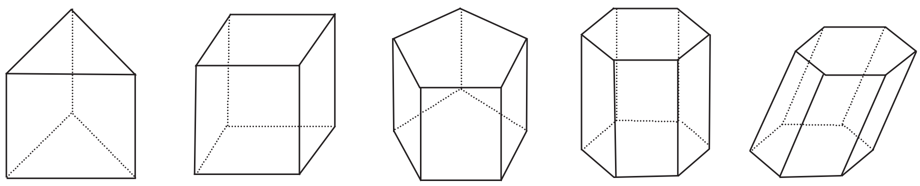
What have you learned?

- A pyramid is a special kind of polyhedron that has triangular side faces.
- A pyramid may or may not have a triangular base.
- When it has a triangular base it is called a tetrahedron.
- The base can also be square like the Egyptian pyramids.
- The first pyramid you investigated is a pentagonal pyramid. It has a base with five sides.
- Any pyramid will have the same number of side faces as the number of sides of the base shape.

4. Prisms

Prisms are another special group of polyhedra. A prism is a 3-D shape which has two polygon faces which are parallel to each other. The rest of the sides are parallelograms. The cereal box you thought about in the first activity was a rectangular prism, or cuboid. A rectangular prism in which all the faces are squares is called a cube.

All of the shapes below are prisms. A prism whose sides are perpendicular to its base is called a right prism. One that leans over is called oblique.



Time needed
60 minutes

Activity 4:
Prisms

Do this activity on separate paper and add it to your portfolio

What you need:
- Paper
- Pencil, eraser, ruler

DICTIONARY:

perpendicular – straight up
oblique – slanted

Work in pairs

1. Look at the shapes above again. Discuss:
   a. What properties does each of these prisms have?
   b. What makes them the same and how are they different?
   c. Discuss and draw rough sketches of the nets you need to build each prism.
2. Work with your partner to design a net for any one of the right prisms that can fit onto an A4 piece of paper. Now build your prism. If it does not work, try again!
3. Once you are satisfied that you have drawn the net correctly and checked that it folds neatly into a prism, practice designing nets for some of the other right prisms shown above.

What have you learned?

- A prism is a special kind of polyhedron whose two end faces are polygons of the same shape and are parallel to each other. All the side faces are parallelograms. The most common type of prism is a cube or a rectangular shaped prism (called a cuboid).
- The number of parallelograms used to build the sides, will match the number of sides in the end face.
- Prisms can be perpendicular or oblique.
- You can make a prism from paper by designing, measuring and drawing a net to include all the faces of the prism.

Time needed
90 minutes

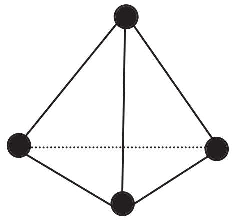
Activity 5:
Polyhedral Skeletons

What you need;
- Toothpicks
- Prestik or jelly tots

Work in pairs

1. You have a lot of experience building polyhedra from nets, that is, a design using the flat faces needed to build a particular solid. Now you're going to have a chance to explore skeletons of the same shapes. You will need a box of toothpicks, some sweets like jelly tots or some prestik.

a. Start by building the simplest regular polyhedron, the tetrahedron. Just stick three toothpicks into three candies to make the triangular base. Then add one more sweet and three more toothpicks to form the tetrahedron.



b. Now try making as many of the other figures you explored in this unit as you can. Use the shapes you made from nets as well as the drawings on page 26 to guide you. Make at least one other platonic solid, one pyramid, and one prism.

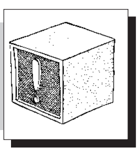
2. Now that you have tried making several of the shapes we have already considered, it's time to get creative. What kinds of shapes do you think are possible to make? When choosing a shape for the face, which shapes seem to be strong? What shapes seem more floppy? Try experimenting and then discuss and share your findings with a partner.

What have you learned?

You can build a skeletal frame of a polyhedron using toothpicks and prestic, sweets, or plasticine, or from straws and string, or similar kinds of materials.
When you build the shapes in this way you will discover which shapes are stronger and which can support others better as you build.

As you work you will think about the different properties of the shapes you use to build the different models. By the end of the activity it should be easy for you to say for example: "I built a cube made of 6 square faces and a tetrahedron made of 4 triangular faces"

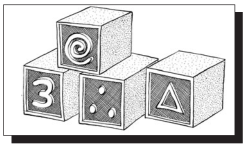
When you have made several different models you should be able to classify them and say if they are one of the platonic solids. or if they are prisms or pyramids or perhaps maybe some of them will just be irregular polyhedra you designed yourself.



Stop and Think

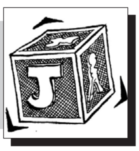
By experimenting with building different kinds of shapes you will notice which are stronger than others. For example a shape built with triangular faces like a tetrahedron, is far more rigid than a cube built with square faces. Have you noticed this in the way roofs (not round ones) are built? Find a room that has a roof that has a prism or pyramidal shape. Look closely at the structure of the wooden or metal parts used to construct the roof trusses or frame.

The reason that triangular frames are used is that the triangle is the most rigid and sturdy of shapes and needs not extra support to keep it in shape, even under pressure. A rectangle or square on the other hand is more "wobbly" and less rigid and would need to have side piece added, to hold its shape when pressure was applied to it.



Linking your learning with your ECD work

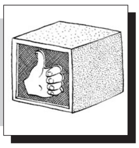
- Building blocks are an essential item in any ECD learning environment. Ideally these should be made from wood or plastic but you can also use different shaped boxes used for packaging or even make some of the shapes yourself, using the ideas for nets you got from working through this unit. Perhaps there is a carpentry workshop in your area and you could ask the people working there to make some blocks for you out off wood off cuts.
- When you have these ready, let the children play freely with them and build whatever they want to. Listen to their language as they work and ask questions that help them to think more about some of the properties of shapes; like how they are the same and how they are different. e.g. by the number of sides and corners they have.
- Prepare a collection of different cardboard boxes that the children can then unfold to investigate the different nets used to build each one. They can then fold them together again, ending with the shape they started with.
- Encourage children to describe and compare the different shapes and classify them, putting the ones that are the same together. Ask questions that encourage them to justify their groupings e.g. "I put all these together because their sides have the same shape" or "These all have pointy ends".



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about polyhedra?
b. How do you think you will be able to improve your understanding of polyhedra?
c. Write down one or two questions that you still have about polyhedra.
d. How will you use what you learned about polyhedra in your every day life and work?

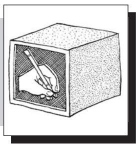


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

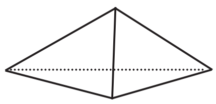
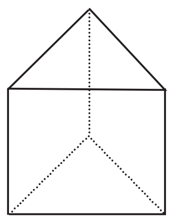
I can:

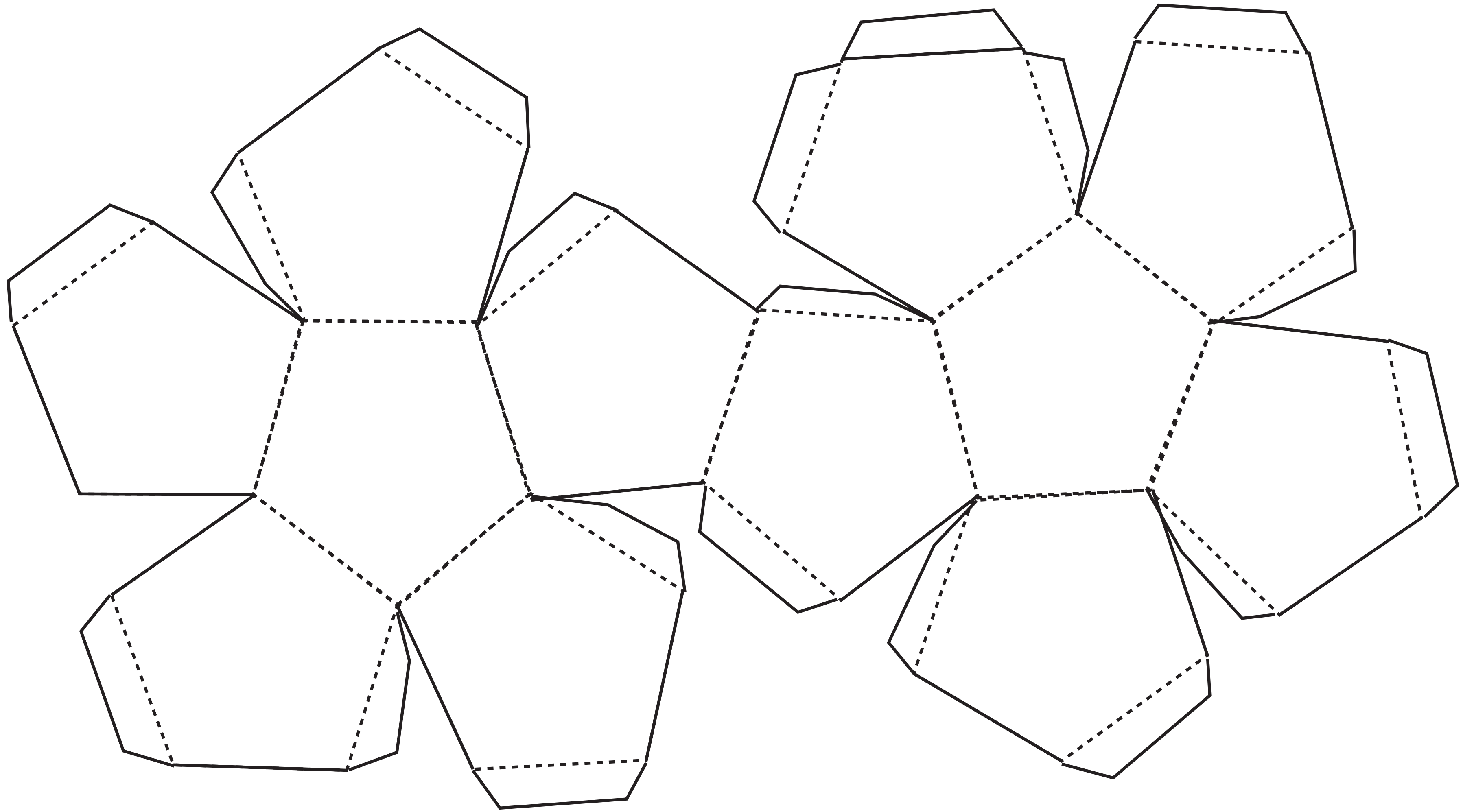
Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Recognize, name, build, draw, and identify the properties of many polyhedra, including:				
Platonic Solids				
Prisms				
Pyramids				
Irregular Polyhedra				



Assignment 3:

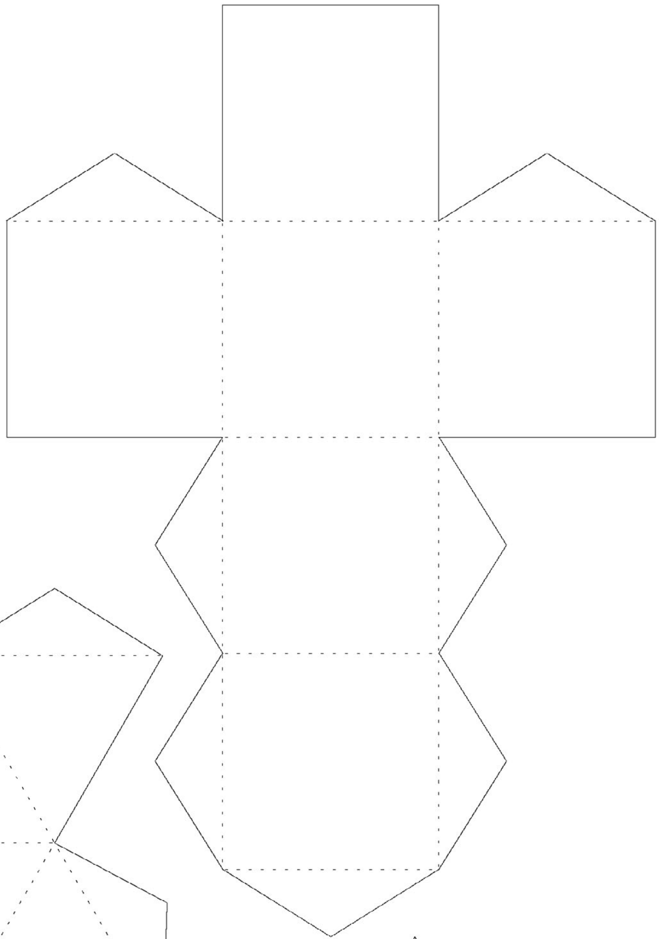
Look around you in your place of work, in your home or when you go shopping. Try and find as many examples of pyramids and prisms as you can. Draw a table like the one below to describe and classify each of the shapes you find. Make a rough sketch of each shape and describe all its properties in detail. Compare tables with a partner's when you are finished.

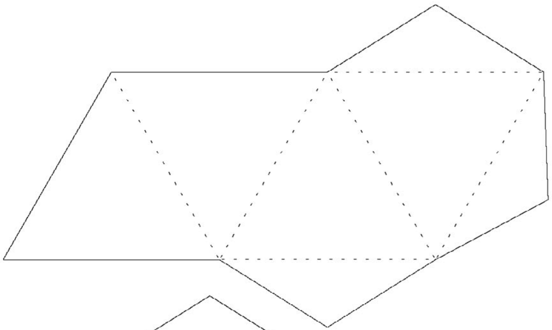
Examples of pyramids and prisms in my world				
	Name of object	**Name of Shape**	**Sketch of the object**	**Properties**
Pyramids	Juice box	Tetrahedron		A pyramid with 4 x triangular faces (tetrahedron)
Prisms	Playing block	Triangular Prism		Two triangular end faces ; four parallel rectangular side faces

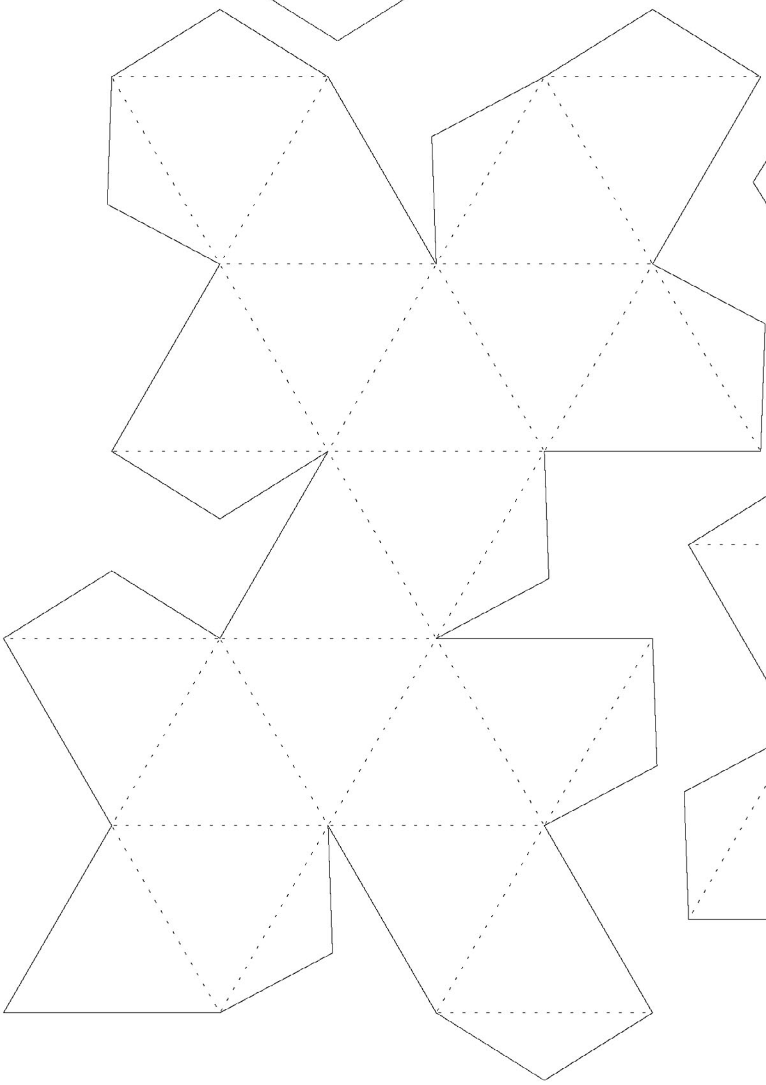


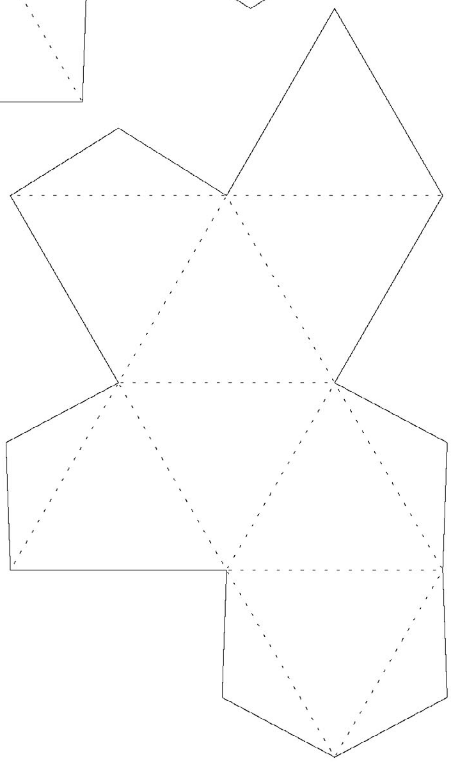
Dodecahedron

Cut out along the solid lines and then score the dashed lines and fold back http://www.math.clemson.edu/~rsimms/neat/deskcal/









UNIT FIVE

Discovering Geometry

In this unit you will address the following:

Unit Standard 7450

SO2:

Use measuring instruments to measure and calculate quantities in various contexts. (Quantities include all of: length, distance, mass, time, temperature, volumes of regular prisms, perimeter, area, weight, surface area, density, displacement and angles. Measuring instruments include all of: rulers, tape measures, scale, clocks, thermometers, capacity measuring instruments, and protractors.

SO3:

Solve measurement problems in various contexts. (practical and non-practical processes, trigonometric right-angled heights and distances.)

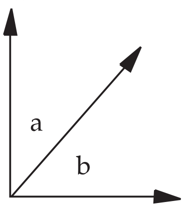
To do this you will:

- learn the definitions of complementary and supplementary angle pairs;
- investigate and understand the basic relationships between angles made by intersecting lines, and by lines that intersect pairs of parallel lines. These relationships are the foundation for solving problems with geometry;
- learn how to demonstrate the value of the sum of the angles in triangles and quadrilaterals.

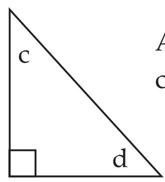
1. Discovering angle relationships

Complementary angles:

If the sum of two angles equals 90° the two angles are called complementary. Two complementary angles combine to form a right angle.



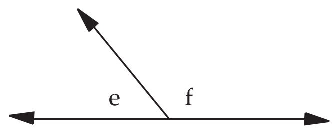
Angles a and b are complementary



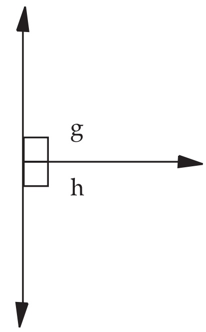
Angles c and d are complementary

Supplementary angles:

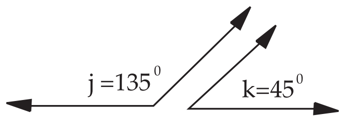
If two angles add up to 180° they are called supplementary angles. If two supplementary angles are combined they form a straight line.



Angles e and f are supplementary



Angles g and h are supplementary



Angles j and k are supplementary

Time needed
70 minutes

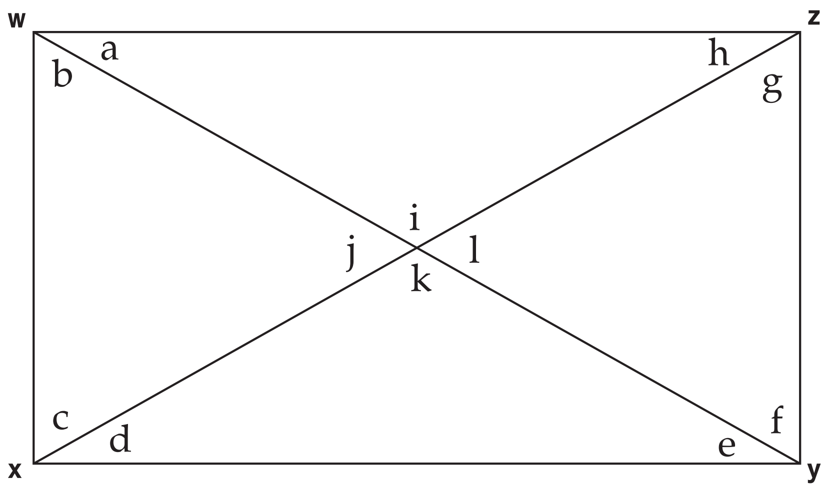
Activity 1:
Find the angles.

Work in pairs

The figure below (WXYZ) is a rectangle.

- Rectangles have opposite sides the same length and four right angles (90°).
- They also have diagonals that are the same length and intersect at their mid-points. You can call the point where the diagonals meet 'C'.

Look at the rectangle carefully and then do the following tasks with a partner. Try to reach agreement on each task.



1. Make a list of all the right triangles. Which ones are congruent (sides and angles the same)?
2. Make a list of all the isosceles triangles (2 sides equal). How do you know they are isosceles triangles? Which ones are congruent?
3. Make a list of all the pairs of supplementary angles.
4. Use the idea that supplementary angles add up to 180° and prove that angles j and l are congruent, and that angles i and k are congruent?
5. Make a list of all the pairs of angles at the corners - a, b, c, d, e, f, g, and h that are complementary angles. Don't stop at 4 pairs! There are a lot more!

DICTIONARY:

intersects – crosses

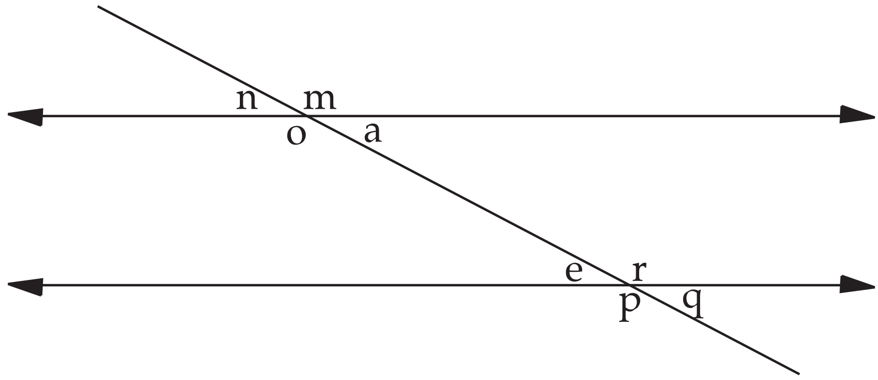
6. Use the idea that complementary pairs add up to 90° to work out which of these angles are the same sizes
7. Considering only the angles at the corners - a, b, c, d, e, f, g, and h – how many different sizes do these angles come in? Group them according to their sizes.

What have you learned?

In Activity 1 you learned that pairs of angles whose sum is 90° are called complementary angles and that pairs whose sum is 180° are called supplementary angles.

Looking again

We can change the picture a little. Take away the right and left sides of the rectangle, and one of the diagonals. Then extend (make longer) the parallel lines of the top and bottom sides of the rectangle, and extend the remaining diagonal. Can you see how you get this picture now?



Activity 2:
Find the angles.

Time needed 20 minutes

Work in pairs

1. Look at the new picture and find all of the pairs of angles that are congruent.
2. Explain how you know they are equal.

What have you learned?

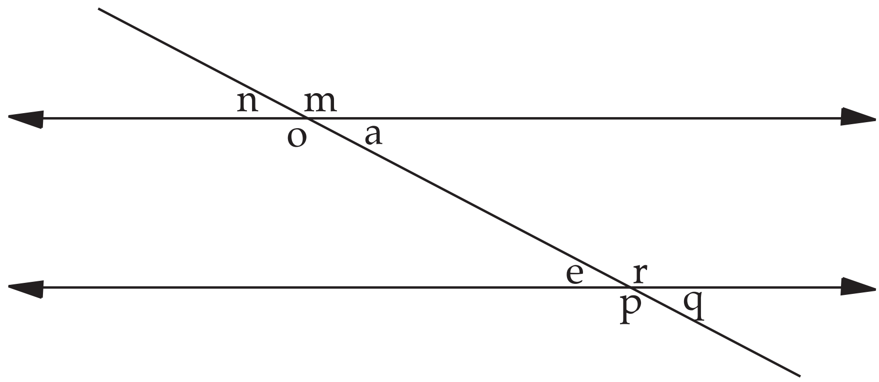
You explored the angles created by a line that intersects a pair of parallel lines. You found that many of the angles are congruent. Here is a summary of the important relationships. Check if you discovered them all.

When you are reading you will also learn the special language mathematicians use to describe these angles.

The line that intersects the two other lines is called the transversal.

DICTIONARY:

interior – inside
exterior – outside



Vertical Angles

- In the diagram above, angles m and o are vertical. So are angles r and p. But so are angles n and a, and angles e and q!
- Vertical angles are not necessarily up and down, they are the angles opposite one another at the intersection of two lines.
- Vertical angles are congruent.

Alternate Interior Angles

- In the drawing above, o and r are alternate interior angles. So are angles a and e.
- If the lines cut by the transversal are parallel, alternate interior angles are congruent.
- They are called interior because they are inside the parallel lines. They are called alternate because they are on opposite sides of the transversal.

Alternate Exterior Angles

- In the drawing above, n and q are alternate exterior angles. So are angles m and p.
- If the lines cut by the transversal are parallel, alternate exterior angles are congruent. They are called exterior because they are outside the parallel lines. They are called alternate because they are on opposite sides of the transversal.

Corresponding Angles

- In the drawing above, angles m and r are corresponding angles. So are a and q, n and e, and o and p.
- They are called corresponding because they are in similar positions in relationship to the transversal and the parallel lines.
- If the lines cut by the transversal are parallel, corresponding angles are congruent.

There are some angle pairs that are not necessarily congruent.

DICTIONARY:

vertices – corners

Consecutive Interior Angles

- In the figure above, angles o and e are consecutive interior angles. They are interior because they are inside the parallel lines. Consecutive interior angles are two angles on the inside of the parallel lines and on the same side of the transversal.
- If the lines are parallel, consecutive interior angles are supplementary. Can you think of a case when they would also be congruent?

Time needed 70 minutes

Activity 3:
How many degrees in all?

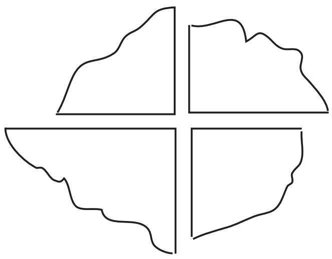
What you need:

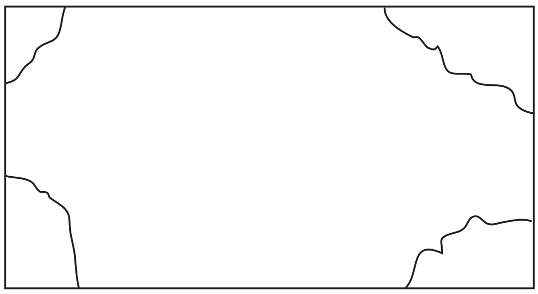
- Paper
- Pencil, ruler

Work alone

There are 360° in a circle.

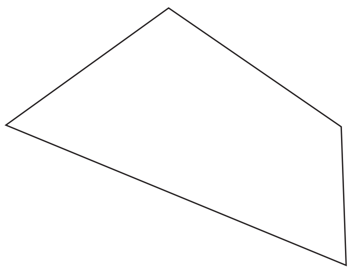
1. Do this exercise to work out what the sum of all the angles in a quadrilateral is.
   - Tear the corners off a rectangle and turn them around to put the points together.
   - You will get 360° - a full circle.





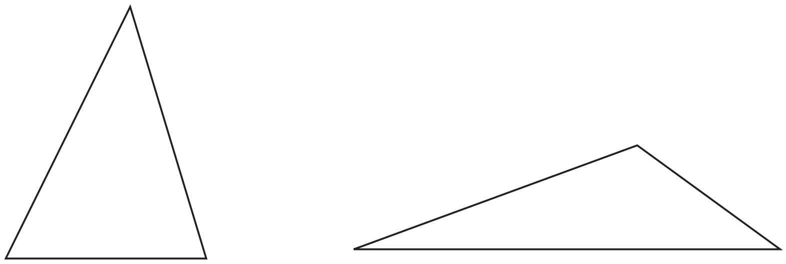
   - In the case of a rectangle, you can find the answer just by thinking about it. It has four right angles, each 90°.

2. What about other quadrilaterals? Try this experiment:
   - From a piece of paper cut out any non-rectangular quadrilateral.
   - Label the vertices A, B, C, D.

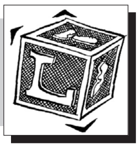


- Then, tear off the corners and put them together.
- What is the result?
- What is the sum, in degrees, of the four vertices in a quadrilateral?

3. Let's ask the same question about a triangle.
   - Draw any triangle, label the vertices.
   - Tear off the vertices and put them together.
   - What is the sum, in degrees, of the angles in this triangle?
   - Try a couple more triangles. Is the sum the same, or different?

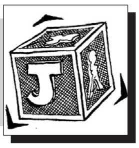


4. What do you think you would get if you tried the same test for other polygons?



What have you learned?

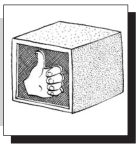
In Activity 3 you learned how to demonstrate that the four interior angles of a quadrilateral sum to 360°, and that the three angles in a triangle sum to 180°.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about angle pairs?
b. How do you think you will be able to improve your understanding of intersecting lines and parallel lines?
c. Write down one or two questions that you still have about demonstrating the value of the sum of the angles in triangles and quadrilaterals.
d. How will you use what you learned about angles in your every day life and work?

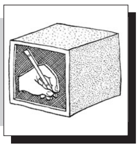


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Learn the definitions of complementary and supplementary angle pairs.				
2. Investigate and understand the basic relationships between angles made by intersecting lines, and by lines that intersect pairs of parallel lines.				
3. Learn how to demonstrate the value of the sum of the angles in triangles and quadrilaterals.				



Assignment 3:

- Fold a sheet of A3 paper to make a square. Then follow the steps on pages 10, 11 and 2 in Unit 2 make a tangram from the square.
- Draw a table like the one below. Based on what you have learned in this unit, enter the following information in the table; the name of each shape, a drawing of each shape; filling in what you what you think the size of both the interior and exterior angles are.
- In the next column write a short sentence to explain how you know this. The last column gives you space to re-draw the shape if you found the angles sizes are different after measuring them with a protractor
- Use a protractor to measure the angles to see if you were correct. If necessary re-draw the shape and show the correct angles sizes in the last column.
- Record your findings in this way for each of the 7 tangram pieces. When you are finished compare your findings with a partner.

Name of shape	Drawing of Shape showing size of interior/exterior angles	How do I know	Corrected drawing
Square		A square is a quadrilateral with 4 equal sides and 4 right angles. Both interior and exterior angles are right angles	

UNIT SIX

Playing with Pythagoras

In this unit you will address the following:

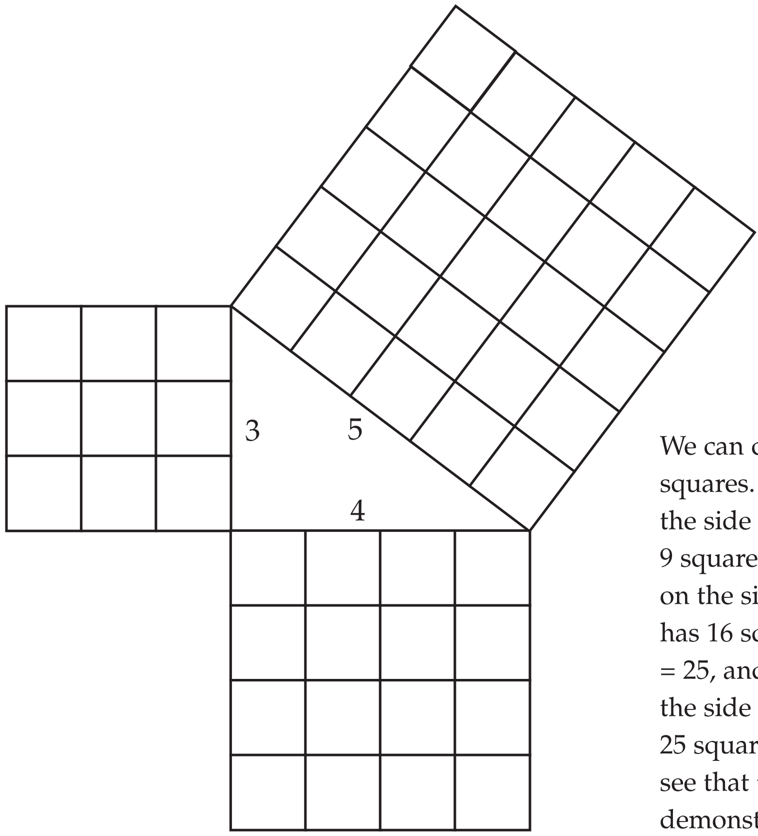
Unit Standard 7450
SO2:
Use measuring instruments to measure and calculate quantities in various contexts. (Quantities include all of: length, distance, mass, time, temperature, volumes of regular prisms, perimeter, area, weight, surface area, density, displacement and angles. Measuring instruments include all of: rulers, tape measures, scale, clocks, thermometers, capacity measuring instruments, and protractors.)

SO3:
Solve measurement problems in various contexts. (practical and non-practical processes, trigonometric right-angled heights and distances.)

To do this you will:
- work with the famous Pythagorean Theorem and learn how to demonstrate it;
- use Pythagoras to find the lengths of sides of triangles to solve problems in real life.

1. Demonstrating the Theorem

The Greek philosopher Pythagoras used a diagram like the one below to show that in a right triangle the area of the square on the hypotenuse (longest side) is the sum of the area of the squares on the other two sides.



We can count the squares. The square of the side of length 3 has 9 squares. The square on the side of length 4 has 16 squares. 9 + 16 = 25, and the square of the side of length 5 has 25 squares, so we can see that the Theorem is demonstrated for this triangle.

We can use this proof for any triangle whose two shorter legs happen to have squares whose sum is the perfect square of another number. But what about other triangles? Do Activity 1 to see how it works for isosceles triangles.

Activity 1: Demonstrating the Pythagorean Theorem using Isosceles Triangles

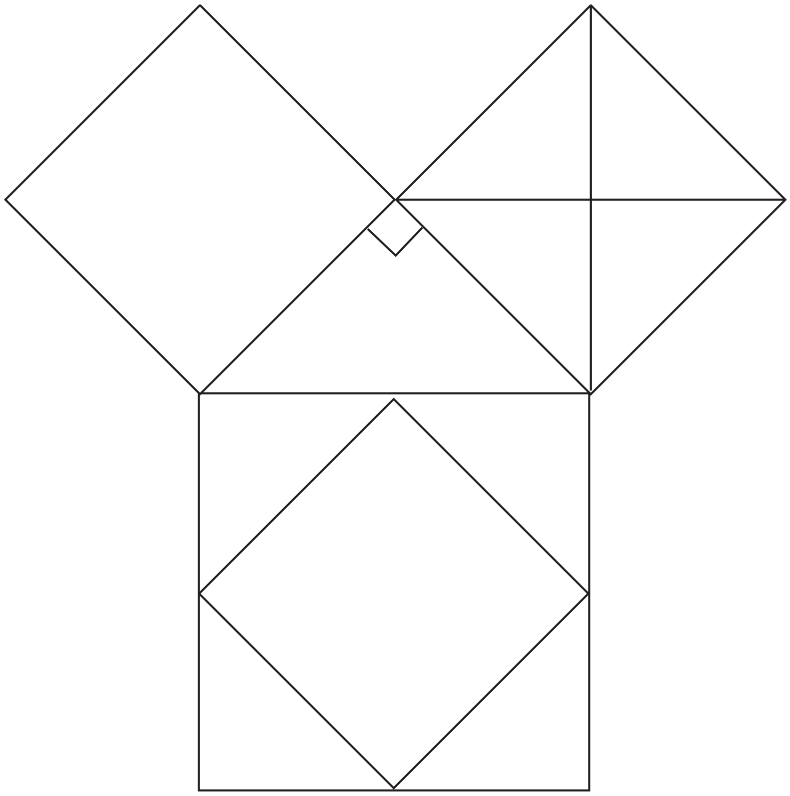
Time needed 20 minutes

Work with a partner

1. Draw an isosceles right triangle. Remember that is a triangle with a right angle surrounded by two equal sides. Use grid paper to make one square for each side.
2. Take the square from one of the smaller sides and cut it along the diagonal in both directions.
3. Now test the Pythagorean theorem for this triangle.
   a. Can you put the smaller square and the four triangles together so that they just cover the larger triangle? If so, you have demonstrated that the area of the square on the hypotenuse is equal to the sum of the areas of the squares of the other two sides!

DICTIONARY:

intersect – cross



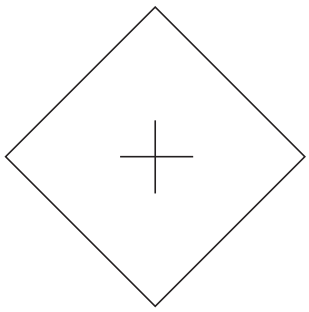
Time needed 50 minutes

Activity 2:
Demonstrating the Pythagorean Theorem for any triangle

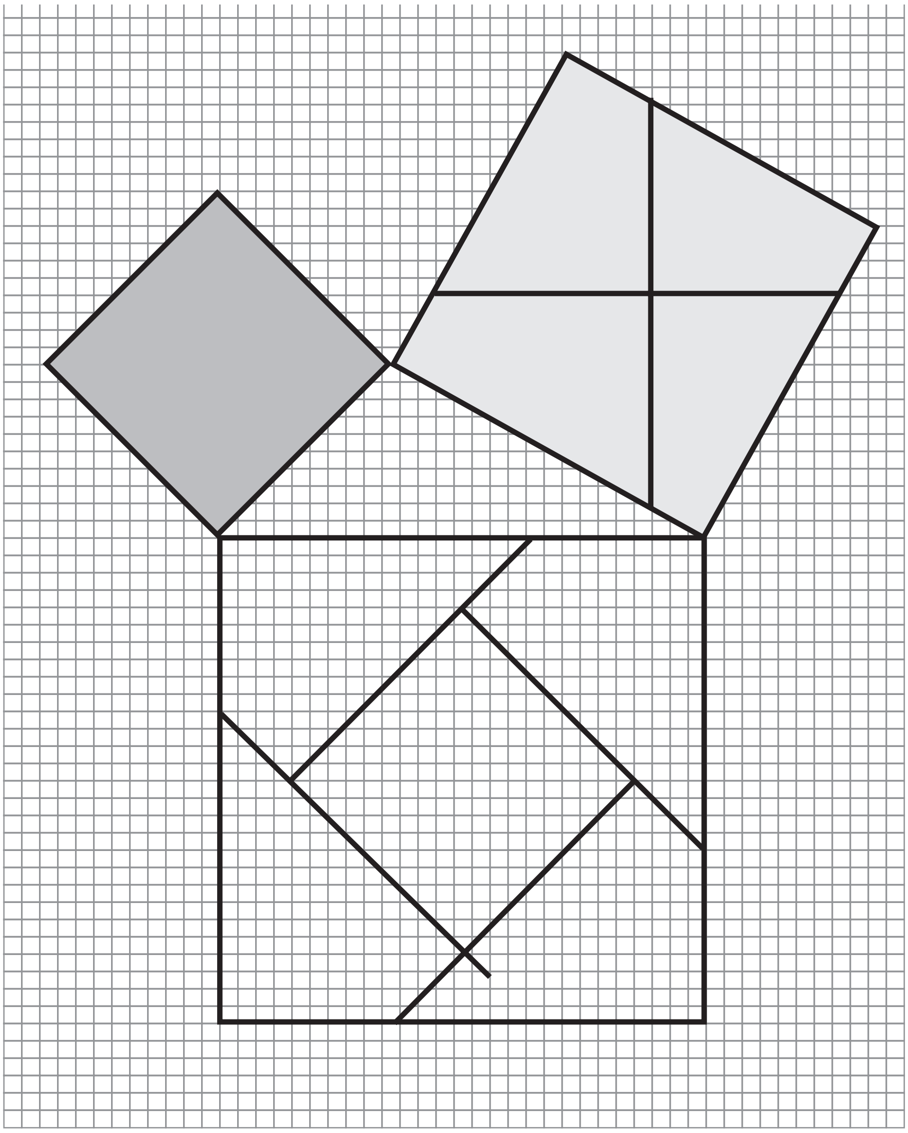
Work with a partner

Now let's try to demonstrate the Pythagorean Theorem for any triangle.

1. Cut out a scalene right triangle from plain paper. One way to be sure you have a right angle is to cut off a corner of a sheet of A4 paper. Using a ruler or grid paper as a guide carefully cut out the squares for each of the three sides, also from plain paper.
2. Now place the triangle on grid paper with the hypotenuse at the bottom. Carefully place the squares next to each side.
3. Find the centre of the mid-sized square. Put a ruler along one diagonal and mark the region near the centre. Then move the ruler to the other diagonal and do the same. The two diagonals intersect at the centre of the square.



4. Through that centre point you make a vertical line. Use the grid paper to help make sure the line is vertical.
5. Now make a horizontal line, again using the grid paper to help you be sure the line is horizontal.
6. Now see if you can put together the small square, and the four pieces of the mid-sized square to exactly form the large square. If you succeed you will have demonstrated the Pythagorean Theorem. Check with others working on the same task to see if their triangles had a square on the hypotenuse, which was equal to the sum of the squares on the other two sides.



Using Pythagoras to find an unknown side of a triangle.

Here is one example of how you can use the Pythagorean Theorem to find the measurements of one side of a triangle if you are given the measurements of the other two sides of the triangle.

If a right triangle has sides of 8cm and 15cm, use the Pythagorean Theorem to find the length of the hypotenuse.

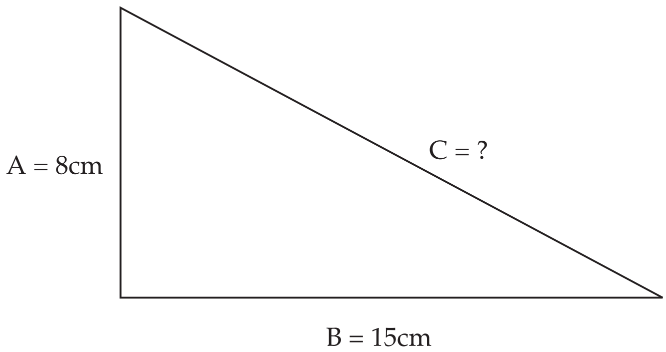
$A^2 + B^2 = C^2$

$(8cm)^2 + (15cm)^2 = C^2$

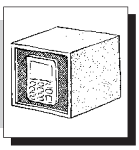
$64cm^2 + 225cm^2 = C^2$

$289cm^2 = C^2$

$17cm = C$

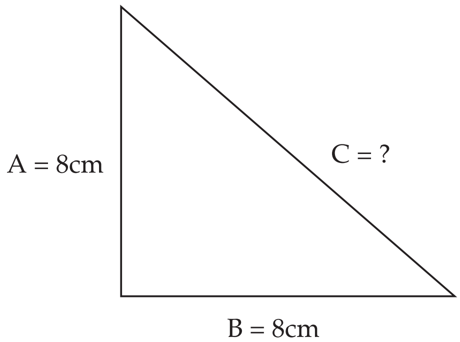


Time needed
30 minutes

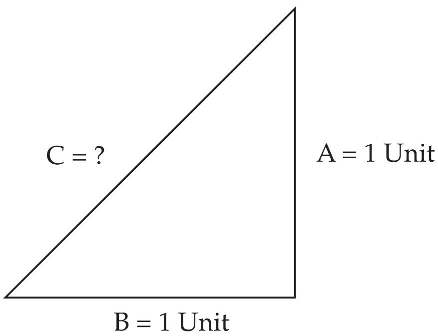


Activity 3:
Find the missing side

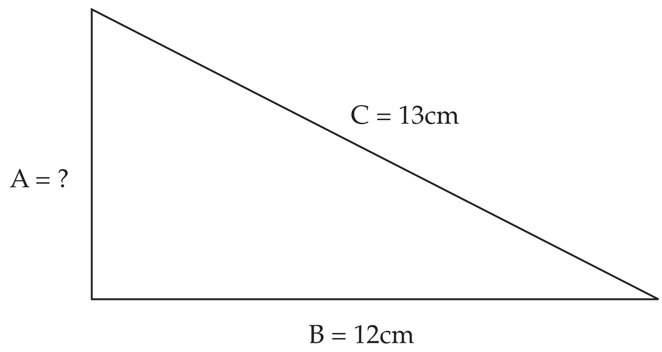
1. Find the length of the hypotenuse, C. Use the √ key on your calculator to find the square root.



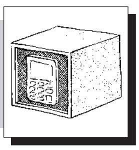
2. Most triangles don't work out to perfect squares. In this problem, use your calculator to find the length of C.



3. In this case we know the length of the hypotenuse and we want to find the length of one of the other sides. How long is side A?



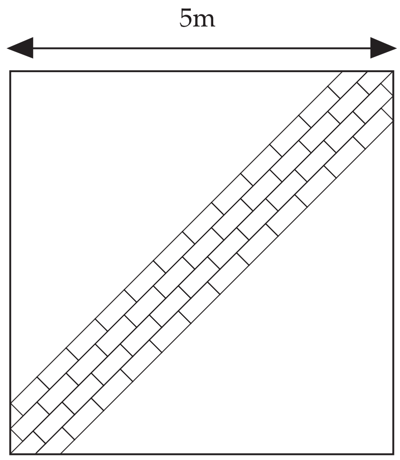
Time needed
30 minutes

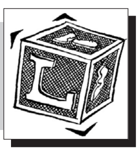


Activity 4:
Solving problems using the Theorem of Pythagoras

1. Imagine you need a ladder to reach a window 4m above the ground. You want to put the foot of the ladder 1.5m away from the wall so that the ladder will be steady when you climb it. Work out how long your ladder needs to be. Do a drawing to help you.

2. You have a square garden, 5m on a side. You want to build a stone path across the diagonal of the garden. How long will the path be?





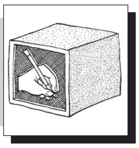
What have you learned?

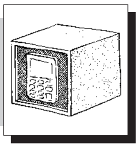
You have learned that the Pythagorean Theorem says that in a right triangle the area of the square on the hypotenuse (the longest side) is equal to the area of the sum of the squares of on the other two sides. You saw how to show that it is true, both by drawing the squares and measuring their areas with grid paper, and by cutting the smaller squares and combining the pieces to form the larger square.

The Pythagorean Theorem can be used to calculate the hypotenuse of a right triangle, or to calculate another side when the length of the hypotenuse is known.

You can also use the Pythagorean Theorem, if you know the lengths of all three sides, to find out whether a triangle is a right triangle. If the sum of the squares of the two shorter sides equals the square of the hypotenuse, it is a right triangle. Otherwise it isn't.

Time needed
90 minutes



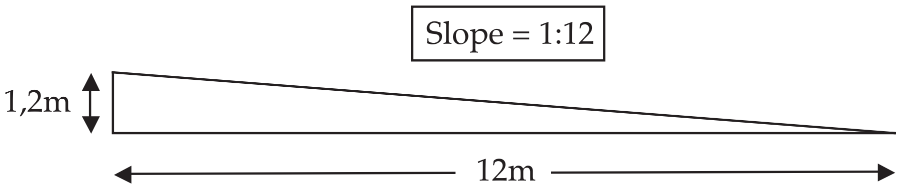


Activity 5:
Design a wheel chair ramp.

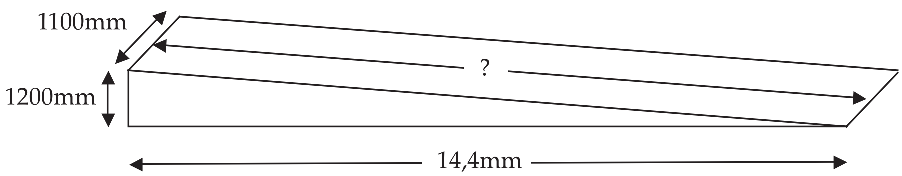
Work with a partner
Do this on separate paper and put it into your portfolio once you are happy with it.

Mrs Maseko wants to build a ramp for her ECD centre to make it accessible to children and adults who use wheelchairs. Luckily, she already has some wood she can use for the frame and the railings, but she'll have to buy some deck boards for the surface. The wood is very expensive, so she needs to calculate the exact amount needed to avoid any unnecessary expense.

The door of the ECD centre is 1 200mm above the ground. She has learned that the ramp must be at least 1 100mm wide, and it can only have a slope of 1:12. That's another way of saying that for every metre you want to go up, the ramp must run 12 metres along the ground.



Part 1:
Mrs Maseko's first thought was that she needed to build a triangular ramp 1 100mm wide, 1 000mm high, and 12 times as long as it was high or 12 x 1 200mm = 14.4 meters long. She was pleased that she knew the Pythagorean Theorem, as this would allow her to calculate how many square feet of decking board she would need. What did she find?



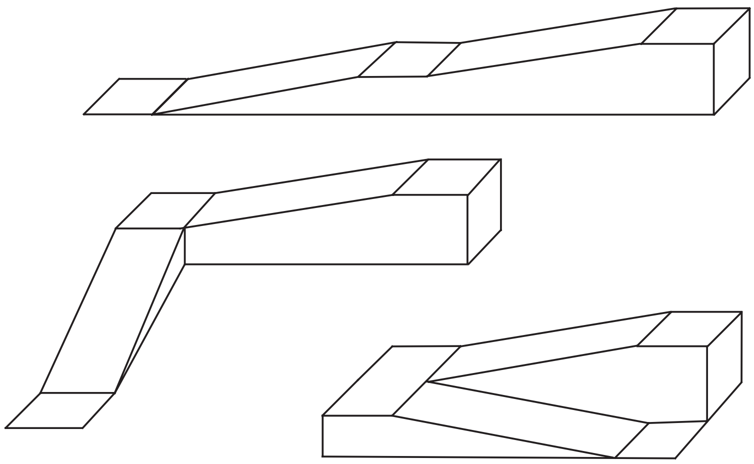
Part 2:
Mrs Maseko thought she should check to see if were any other things she needed to know about wheelchair ramps before she began.

She learned that she needs some landings. The landing at the bottom has to be at least as wide as the ramp, and at least 1 200mm long.

The one at the top has to have room for the person to get through the door. The exact dimensions depend on the design of the ramp and how the door swings. If the door swings out, onto the ramp, the landing has to be 1 200mm, plus the length taken up by the door swing. If the door swings in, away from the ramp, then the landing can be 1 500mm. Since the door at the ECD centre swings in, Mrs Maseko plans to add a top landing of 1 500mm.

She also learned that a ramp that rises more than 1 metre, like this one, needs a landing somewhere in the middle, so the wheelchair user can rest. That landing has to be at least 1 200mm long, and as wide as the ramp.

She looked at designs of some ramps and found that some are straight, some are l-shaped, and some zig-zag. She saw that if she wanted to zig-zag she would need a bigger landing. She also learned that she needed a handrail on both sides of the ramp.



There was a lot to think about. Mrs Maseko asked her colleagues to help her figure out how to design a ramp that met all the requirements. See if you can help her.

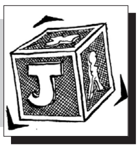
Make a sketch of the ECD Centre (you can make it look any way you like, just remember that the door is 1 200mm above the ground.) On the scale drawing (or you can make a scale model, if you prefer), show the ramp, landings, and handrails – everything that the specifications require. Discuss how you will build the framework under the ramp. Try to reach consensus on what you will have to do to make it strong.

And finally, use the Pythagorean Theorem to calculate how much decking board you'll need for the ramp and landings! Give your answer in mm^2 or m^2.

Part 3:
Mrs Maseko has found deck boards for sale that are 10cm wide. How many linear metres does she need?

It is sold in lengths of 2m, 3m, and 4m. If she wants to build the ramp with no seams – that is, with only whole boards, 1 100mm each, across the ramp,

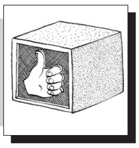
- which length board should she choose,
- how many pieces will she need,
- how much wood will be left over?



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about Pythagoras?
b. How do you think you will be able to improve your understanding of Pythagoras?
c. Write down one or two questions that you still have about Pythagoras.
d. How will you use what you learned about Pythagoras in your every day life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Work with the famous Pythagorean Theorem and learn how to demonstrate it.				
2. Use Pythagoras to find the lengths of sides of triangles to solve problems in real life.				

UNIT SEVEN

Area and perimeter of rectangles

In this unit you will address the following:

Unit Standard 7450
SO1:
Apply relationships between common quantities in various contexts. (Mass and weight, distance and displacement, speed and velocity, volume and density, volume and surface area, area and perimeter, distance and time, volume and capacity.)

SO2:
Use measuring instruments to measure and calculate quantities in various contexts. (Quantities include all of: length, distance, mass, time, temperature, volumes of regular prisms, perimeter, area, weight, surface area, density, displacement and angles. Measuring instruments include all of: rulers, tape measures, scale, clocks, thermometers, capacity measuring instruments, and protractors.)

SO3:
Solve measurement problems in various contexts. (practical and non-practical processes, trigonometric right-angled heights and distances.)

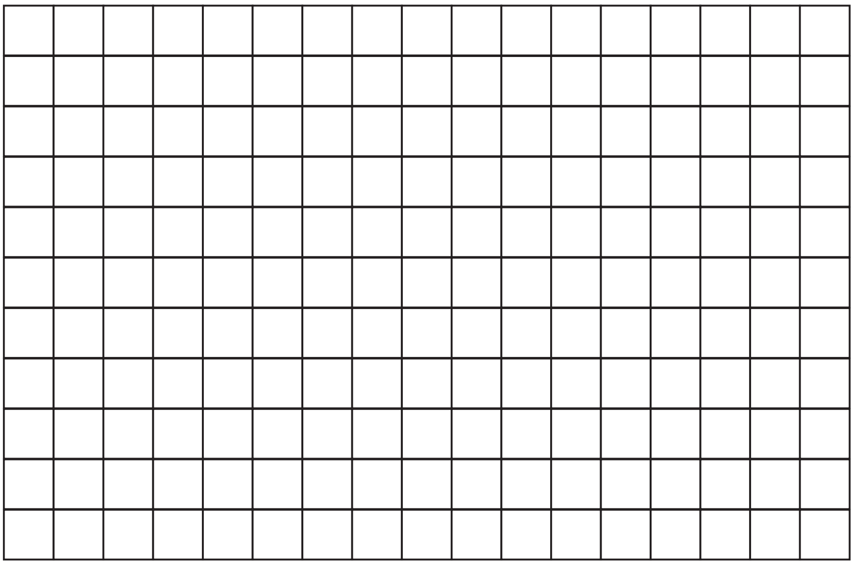
To do this you will:
- calculate the area and perimeter of squares and rectangles using the standard formulae;
- explain relationships between perimeter and area;
- apply your knowledge of area and perimeter to everyday situations.

1. Revision

Activity 1:
Revising what you know

Time needed
10 minutes

Work in Pairs
On the square grid below draw six different shaped rectangles.
Count the number of squares that cover the rectangles to find their area.



Stop and Think

Is there a quicker way to find the areas of your rectangles?

2. Ways to measure area

You probably found out in Activity 1, that the quick way to find the area of a rectangle is to multiply the length by the breadth or width. So we can say that the formula (rule) for finding the area of a rectangle is $l \times b$, or $l \times w$.

We use different sized square metric units to measure different sized areas:

- Very small areas are measured in 1mm by 1mm squares – one square millimetre or mm^2
- Small areas are measured in 1cm by 1cm squares – one square centimetre or cm^2
- Larger areas are measured in 1m by 1m squares - one square metre or m^2
- Land or farms are measured in 100m by 100m squares called hectares, written as ha. One hectare is 10 000m^2 (ten thousand square metres)
- Very large areas are measured in kilometre squares, written as km^2

Look at this interesting information:

- One square kilometre is the same as 1 000m × 1 000m which is 1 000 000m^2 which is one million square metres!
- There are 100 hectares in 1 square kilometre.

Activity 2:
Measuring square areas

Time needed
40 minutes

Work Alone

Use the formula for calculating area to solve these problems. Give your answer in the correct unit each time.

1. A local council has donated a piece of land for a health centre to be built. The land measures 28 metres long and is 32,5 metres across. What is the area of the piece of land?
2. Thabo is going to paint the roof of the Bantwana Bami ECD centre. He has to work out how many tins of paint he should buy. He has this information:
   - The label on the tin says that a tin of paint covers an area of $4m^2$
   - The price of the paint is R278.
   - The roof is in the shape of a rectangle which is 17m long and 6m wide.

Use this information and some quick calculation methods to find out the following:

   a. The number of tins of paint that Thabo will need.
   b. What the paint will cost.

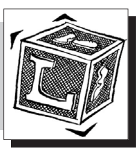
3. A local government department wants to buy an area of 12, $5km^2$ of land for a housing development project. The present owner wants R8 600 per hectare for the land. Use this information and your knowledge of working with decimals to help you work out:
   a. How many hectares there are in $12{,}5km^2$?
   b. What will this much land cost the government to buy?
4. Discuss and compare your methods of doing questions 1-3 with a partner or with a group of colleagues. You can use a calculator to check your answers. If your calculations were wrong, try to find your mistakes and correct your work.
5. Practice converting these measurements:
   a. Express each measurement below in hectares e.g. $23{,}5km^2$ = 2350ha

      (i) $34{,}76km^2$ (ii) $3{,}208km^2$ (iii) $435km^2$ (iv) $5{,}67km^2$ (v) $124{,}5km^2$

   b. Express each measurement below in square kilometres e.g. 123ha =$1{,}23km^2$

      (i) 346,7ha (ii) 500ha (iii) 899,8ha (iv) 987.56ha (v) 4 587ha

   c. Use a calculator to check your answers.
   d. Compare answers with a partner.



What have you learned?

When you measure the amount of space that something takes up you are measuring area. To calculate area you multiply the length of a space by its width or breadth, and give your answer in square units.

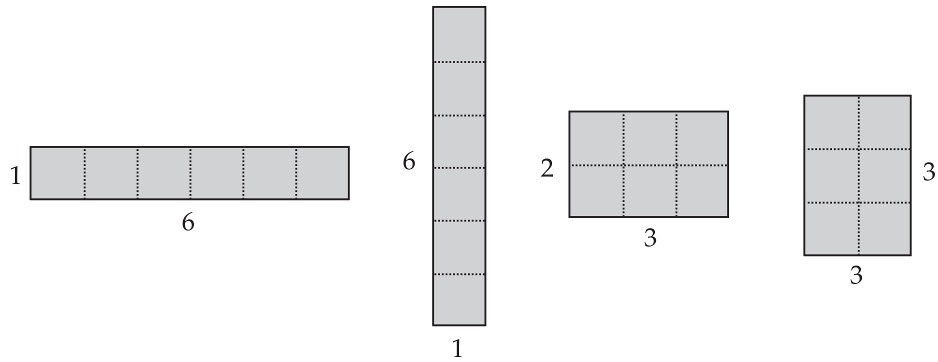
There are some units in the metric system for measuring area that you use more often than others. For example you generally use square millimetres, centimetres and metres. You do not often use square decimetres (10 square centimetres) or decametres (10 square metres).

In addition to square kilometres, you can use hectares to measure larger land sizes like the size of a farm. One hectare is the same as 10 000 square metres or one hundredth of a square kilometre.

Time needed
15 minutes

Activity 3:
Same areas, different perimeters

Work in pairs
You now know that you can find the area of a rectangle by multiplying the length by the breadth - l × b. Look at the diagram below. You can see that different-shaped rectangles can have the same area. all the rectangles below have an area of 6cm^2.



6 = 6 x 1 **6 = 1 x 6** **6 = 3 x 2** **6 = 2 x 3**

1. On grid paper draw sketches to show all the possible rectangles with the following areas (in cm^2) and complete the table.

Area	Different lengths and widths:	Number of rectangles
cm^2	1 x 6; 6 x 1; 2 x 3; 3 x 2	4
7 cm^2		
8 cm^2		
9 cm^2		
12 cm^2		
20 cm^2		
36 cm^2		
37 cm^2		
60 cm^2		

2. Discuss the links you can see between number and area.

What have you learned?

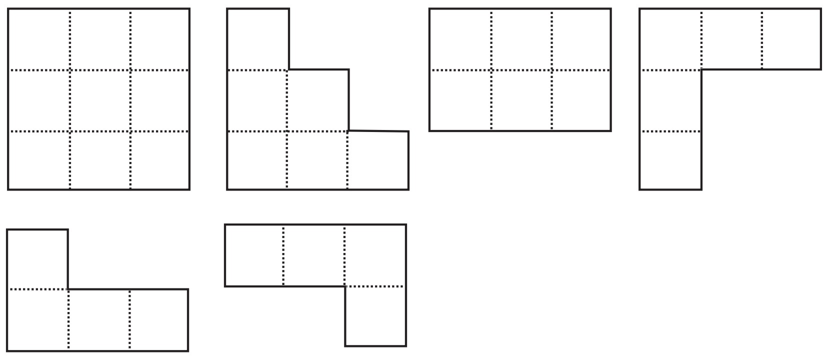
You can see that different shapes can have the same areas. You also noticed that shapes with the same areas may have different perimeters.

Activity 4:
Same perimeters; different areas

Time needed
20 minutes

Work Alone

1. Count how many steps around each of these shapes cover. Do you agree they all have the same perimeter? Are they equally big? How big is each house? Which house is the biggest? Write a short summary of what your answers tell you.



Work with a partner

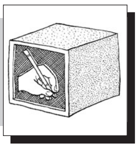
2. The mothers of the Bantwana Bami ECD Centre raise chickens as a source of income for their Centre. They want to fence off two areas to make two chicken runs. They want the runs to be rectangular in shape and to cover the largest possible area. They also do not want their runs to be too narrow. They have 48 metres of fencing they can use.
   a. How long and how wide should they make the run? Complete a table like the one below to show the possible different lengths and widths to help you decide.

Length (m)	Width (m)	Area (m^2)
10(x2)	14(x2)	140m^2

What have you learned?

In the previous activity you discovered that shapes of the same areas can have different perimeters. From the investigation above you will have noticed that shapes with the same perimeters can have different areas.

Time needed
20 minutes

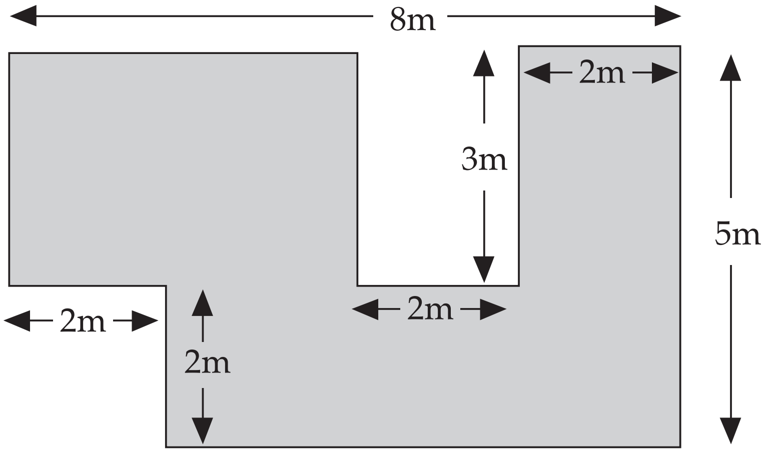


Activity 5:
Using perimeter and area in the work place.

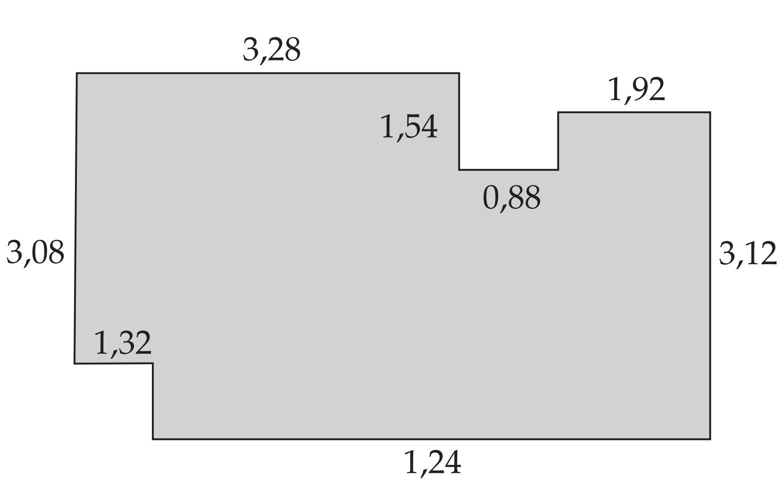
Add this activity to your portfolio. Show all your calculations.

Work with a partner

Sometimes you are given an area of a shape to work out that is almost like a square or rectangle but has a piece missing like this.



1. With a partner think about how you can use the measurements you are given to calculate the perimeter and area of the shaded part of this figure.
2. Now calculate the area and perimeter of the shape.
3. The Bantwana Bami ECD Centre is improving their building by fixing fascia board all around the edge of the roof. The fascia board is sold in lengths of 2,4m, at R129.00 per 2,4m length. A rough sketch of the roof, as seen from above, is given below. Read the plan carefully then answer these questions.



a. How much fascia board will they need?
b. What will it cost?
c. What is the area of the total floor space?
d. The cost of linoleum is R68 per square metre. What will it cost to cover the floor?

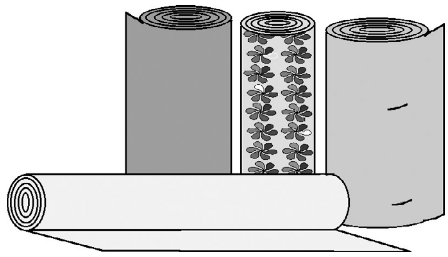
Time needed
30 minutes

Activity 6:
Buying a carpet

Work alone

The Bantwana Bami ECD Centre wants to buy a carpet for the new toy library. They have a limited budget to spend on carpeting. The Toy Library Committee finds out that the carpets come in rolls 3m wide and are sold per running metre. Different types of carpets have different prices, as follows:

Carpet	Price per metre
Pebbles	**R185**
Nora	**R249**
Afrika	**R299**



These prices include VAT. They must also buy underfelt at R25 per metre. The labour to lay the carpet costs R12 per square metre.

Ms Mahlathini prefers the Pebbles design and Mr. Mabena prefers the Afrika design. Mr. Pieterse likes the Nora design.
Discuss the following questions and do the tasks.

1. What will they need to find out to see if they can afford to buy any of these carpets? Write down some ideas.
2. Back home they measure that the library is 5,23m long and 4,52m wide. What is the area of the room?
3. Draw a sketch to show how you would cover the area of the room with the carpeting. Work out how many running metres you would need to buy.
4. Estimate roughly what it will cost to carpet the room using each of the three designs. Estimate cost of the underfelt. Add the two costs together.
5. Compare your sketches and your cost estimations with a partner.
6. Use a calculator to find the exact costs. Compare these with your estimations.
7. The committee chooses the Nora design. Once the carpet is laid, they decide to put skirting board around the edges. This costs R33 per metre.
   a. How much skirting board will they need?
   b. What will it cost?
8. What is the total cost of carpeting the library?

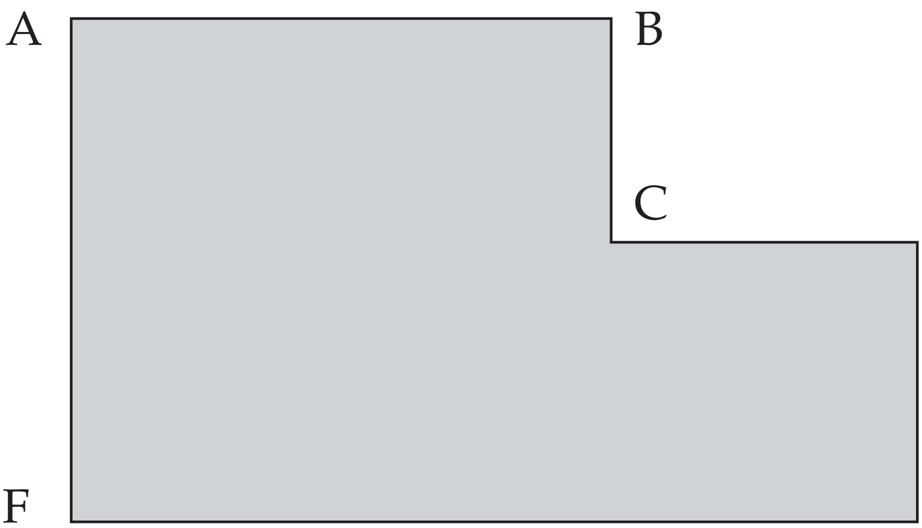
Time needed
30 minutes

Activity 7:
Tiling a bathroom

Work alone

Bantwana Bami decides to use funds to tile the kitchen. Read all the information below and then do the tasks.

The kitchen is L-shaped like this:



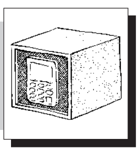
The distance:

- from A to B is 3,21m
- from B to C is 2,34m
- from C to D is 2,98m
- from A to F is 6,41m.

Two sizes of floor tiles are available:

- 9,7cm by 19,7cm
- 19,7cm by 29,9cm.

The smaller tiles cost R495 for 100 and the larger tiles cost R665 for 100.



1. What will it cost to tile the room with the small tiles?
2. What will it cost if they use the big tiles?
3. Check your answers with a partner. Discuss, compare and evaluate the different ways you have used to do the calculations. Discuss which ways were easier to follow and why.

What have you learned?

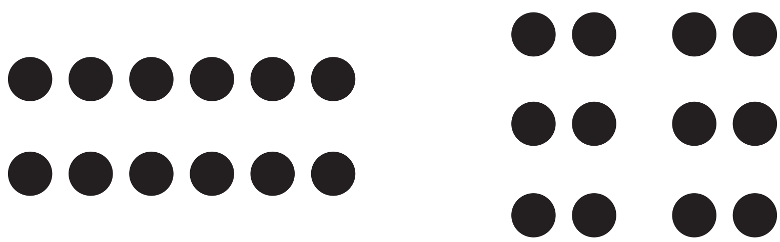
In everyday life you often have to calculate the perimeter and area of spaces. Maybe you have to buy a carpet or buy ceiling materials. If know the measurements of the room and the materials you can estimate and then calculate how much material you need and what it will cost. You can then decide if you have the budget to do the job.

Some goods are charged per length, like edge-to edge carpeting or dress cloth or skirting board. For these you also need to know how wide the materials are. Others like wood or tiles are sold per square metre.

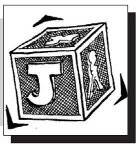
It is useful to be able to estimate costs in your head as well as using calculators. This teaches you skills such as rounding off and compensating.

Linking your learning with your ECD work

- You learners can begin to explore area comparing the size of similar objects like shapes, blocks, leaves etc. They can group similar objects in order of size from biggest to smallest; smallest to biggest.
- Older children can arrange a set number of counters into different sized rectangles and patterns. In this way they will be working informally with multiples and factors. Encourage them to use their own informal language to explain what they do, such as:
  "This is 6 put out 2 times." (left hand side) or "This is 2 put out 6 times." (right hand side).



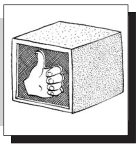
- Older children can trace around objects on square grid paper and count how many blocks the different outlines cover. To make it simpler they can count half-blocks as wholes and do not count blocks less than half.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about area and perimeter of squares?
b. How do you think you will be able to improve your understanding of the formula to calculate the area and perimeter of squares and rectangles?
c. Write down one or two questions that you still have about the area and perimeter of squares and rectangles.
d. How will you use what you learned about area and perimeter in your every day life and work?

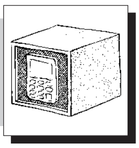


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Calculate the area and perimeter of squares and rectangles using the standard formulae				
2. Explain relationships between perimeter and area				
3. Apply your knowledge of area and perimeter to everyday situations				



Assignment 4: Measuring your space

Measure the area and perimeter of all the rooms in your ECD centre or your house, except for the kitchen and the bathroom.

1. Calculate the following:
   a. The amount of tiles you will need to cover the entire floor space with tiles that measure 60 × 60cm.
   b. If the tiles cost R129.50 a square metre, what will this cost?

2. If you want to carpet the entire floor area with carpeting that costs R156 per square metre but is on sale for less 15%. How much will you pay?

3. You want to put skirting board around the edges of the carpet.
   a. How much will you need?
   b. If skirting board comes in 3,6m strips and each strip costs R45 per square metre, how many strips will you need?
   c. What will this cost you?

UNIT 8

Perimeter and Area of other Polygons

In this unit you will address the following:

Unit Standard 7450
SO1:
Apply relationships between common quantities in various contexts. (Mass and weight, distance and displacement, speed and velocity, volume and density, volume and surface area, area and perimeter, distance and time, volume and capacity.)

SO2:
Use measuring instruments to measure and calculate quantities in various contexts. (Quantities include all of: length, distance, mass, time, temperature, volumes of regular prisms, perimeter, area, weight, surface area, density, displacement and angles. Measuring instruments include all of: rulers, tape measures, scale, clocks, thermometers, capacity measuring instruments, and protractors.)

SO3:
Solve measurement problems in various contexts. (practical and non-practical processes, trigonometric right-angled heights and distances.)

To do this you will:
- find the perimeter of triangle and other regular and irregular polygons;
- find the area of triangles and other regular and irregular polygons;
- demonstrate that the area of a triangle is half the area of the rectangle that just fits around it.

1. Remembering perimeters

You remember that perimeter means distance around an object. You learned how to measure the perimeter of squares and rectangles. Now you are going to find the perimeter of other polygons by finding the total length of all their sides.

Activity 1:
More perimeter investigations

Time needed
30 minutes

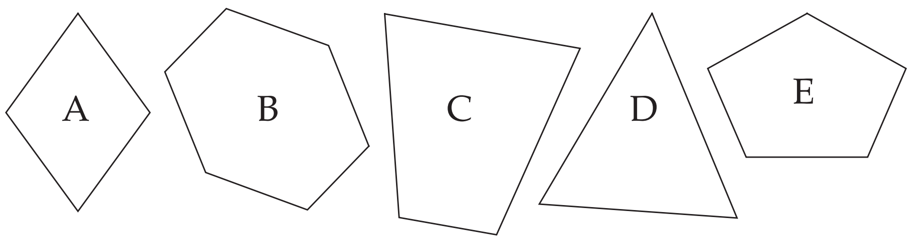
Work alone
Follow these steps:
1. Draw a table with headings like the one below.

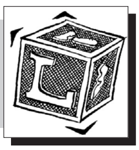
Number of shape	Number of sides and angles	Name and description of shape	Estimated Perimeter	Measured perimeter
A	4	Diamond/Rhombus Also a quadrilateral	1.5 + 1.5 + 1.5 + 1.5 = *6cm*	

2. Start with shape A. Describe the shape in as much detail as possible (see example).
3. Estimate the perimeter (the combined length of its sides).
4. Write your estimate in the correct column.
5. Now measure the sides of the shape with a ruler to find the actual measurements in centimetres or millimetres. Fill this answer in the correct column.
6. Compare your estimate with the real measurements.

Work alone

7. Share your findings with a partner. Check one another's measurements and discuss the ways you described each shape. Look to see if your estimations improved over time!





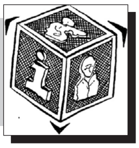
What have you learned?

In this activity you realized that you can find the perimeter of any polygon if you measure the length of each side and add these together. If the sides are the same length you only find the length of one side and then multiply this by the number of sides. In the rhombus above this will be $1.5 \times 4 = 6$cm.

You recorded and organized your information in a detailed table. This gave you an opportunity to practice your data skills as well. You can see how using tables make it easy to work with lots of information at the same time. If you work in a neat and orderly way it is easy to analyse and compare your findings with someone else.

Tracing around shapes

Make cardboard shapes like the ones you worked with in the table in Activity 1. Divide an A4 page into 2cm blocks. Children place a shape on the grid. Children first predict and then count to see how many blocks each shape covers. Blocks that are more than half covered count as 1. Blocks that are less than half covered are not counted.



Time needed
60 minutes

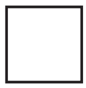
2. Area of triangles

Activity 2:
Area of triangles

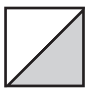
Work with a partner

Work through the following steps carefully. Stop and think often. Ask questions and discuss with peers or your trainer. Take as much time as you need.

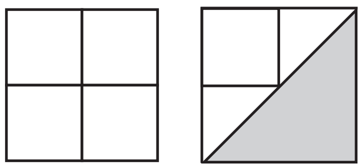
Now that you know how to find the area of a rectangle you can change the method a little to find the area of other shapes as well.

Look at this 1cm square. 

The area of this square is 1cm × 1cm. Right?
If we draw a diagonal and separate the square into two equal halves, each triangle has an area of $\frac{1}{2}$ cm^2.



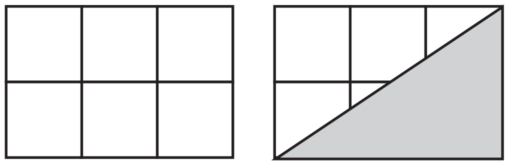
Think about a square with sides of 2cm. Count the squares and you will see that the area of the square is 4cm^2.



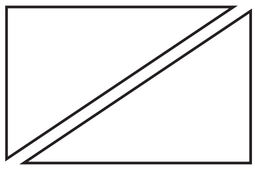
Count the squares in the triangle. There is one whole square and two half-squares. The area of the triangle is $1 + \frac{1}{2} + \frac{1}{2} = 2$cm^2.

Again, the area of the triangle is half the area of the square.

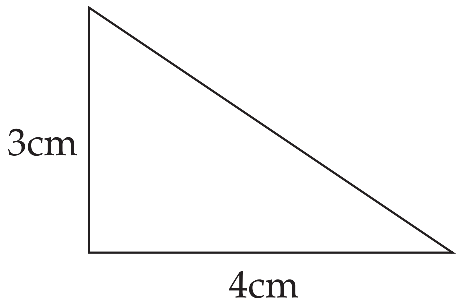
Does this work for rectangles that are not squares?



Here the area of the rectangle is 3cm × 2cm = 6cm^2. The triangle is made up of one whole square and some part squares. It is hard to say the exact area of the triangle just by counting squares. But we know that the diagonal divides the rectangle in two equal triangles. If we rotate one triangle it will exactly cover the other.

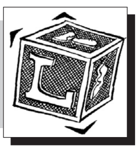


The area of the rectangle is 3cm x 2cm = 6cm^2. The area of the triangle is half that much, or 3cm^2.



Try it yourself.

What is the area of triangle?



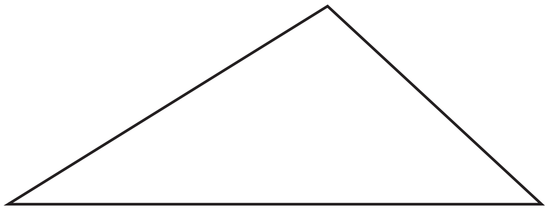
What have you learned?

You have learned how to find the area of a certain type of triangle that forms part of a rectangle.

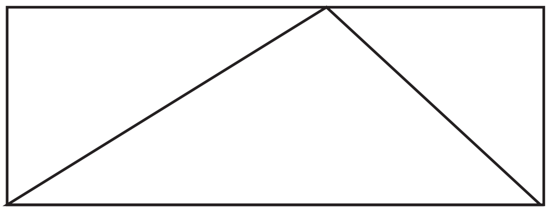
But these are not the only kinds of triangles you get. So now you can investigate how to find the area of other kinds of triangles.

Finding the areas of other kinds of triangles

Let's start with a scalene triangle



Put a box around it.

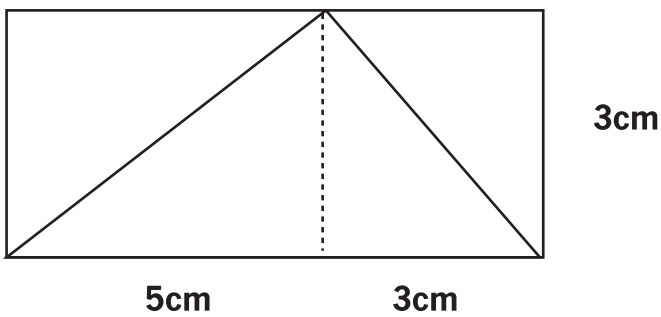


To find the area of the box you need to know the lengths of the sides of the rectangle. The top and bottom of the rectangle are 5cm. This is the same length as the base of the triangle.

The right and left sides of the rectangle are 3cm. This is the same as the height of the triangle.

So, the rectangle has an area $A = 3\text{cm} \times 5\text{cm} = 15\text{cm}^2$

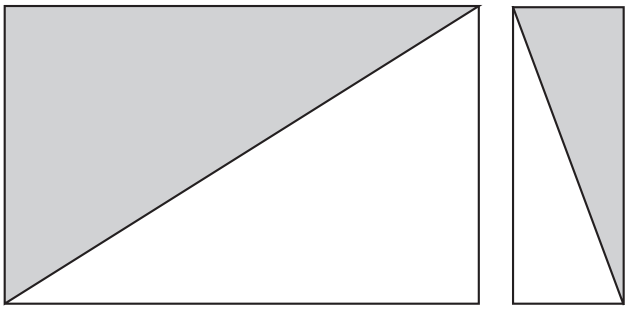
But what is the area of the triangle?

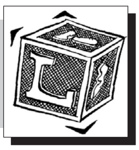


You can see that the $3\text{cm} \times 5\text{cm}$ rectangle is made up of two smaller rectangles. Each of the parts is half covered by the triangle.

Since the triangle covers half of each part of the rectangle, its area is half of the rectangle's area.

Once again we have found that the area of the triangle is $A = \frac{1}{2}$ (base × height)
In this case:
$A = \frac{1}{2} \times (5cm \times 3cm)$
$= \frac{1}{2} \times (15cm^2)$
$= 7{,}5cm^2$



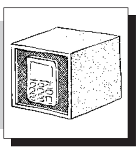


What have you learned?

That was quite an exercise! You can see now how the area of a triangle is half the area of the rectangle that just fits around it. From all these activities you have discovered a rule about the area of triangles. In mathematics this is called a formula. The formula for the area of a triangle is
$A = \frac{1}{2}$ (Base × Height)

Now you can apply what you have learned.

Time needed
15 minutes

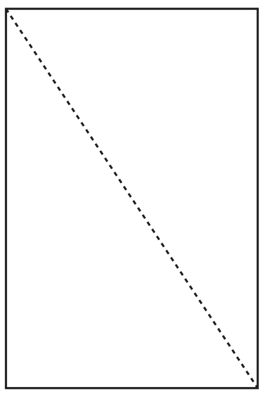


Activity 3:
Practice what you've learnt.

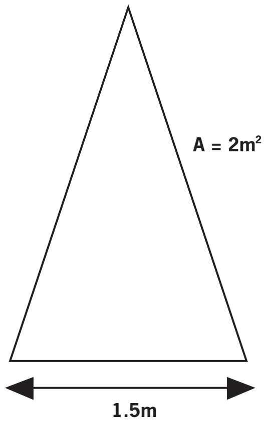
Work alone

Do the following exercises:

a. The dimensions of A4 paper are 209,9mm × 297,0mm. If you cut a sheet along a diagonal what is the area of each of the triangles?

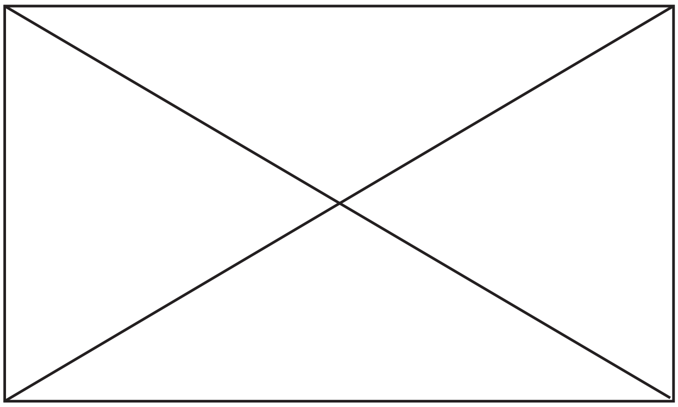


b. You can use the same relationship to find the base or the height if you know the area. This has an area of $2m^2$. Its base is 1.5m. What is its height? (Use your calculator. Give the answer to 2 decimal places.)

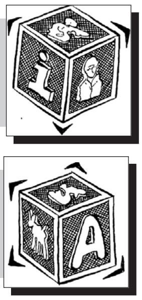


c. Cut out a non-square rectangle about the size of a postcard. Carefully draw in the two diagonals. You have made four isosceles triangles. The top and bottom triangles are congruent. They have the same sides and angles, and obviously they have the same area. The left and right triangles are also congruent and also have the same area.

d. Do you think the area of the top and bottom triangles is the same as the area of the left and right triangles?



There are a number of ways to solve this riddle. Think about it and talk about it with someone. Cut out the triangles and experiment. Maybe you can find an answer without cutting. Have fun!



Time needed
60 minutes

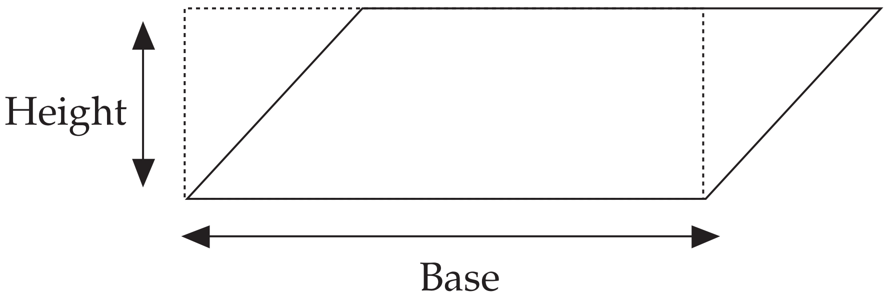
3. Area of other polygons

Activity 4:
Area of Other Polygons

Work with a partner
Read and work through the text below. Stop and ask yourself questions or ask your trainer, if there are things you don't follow.

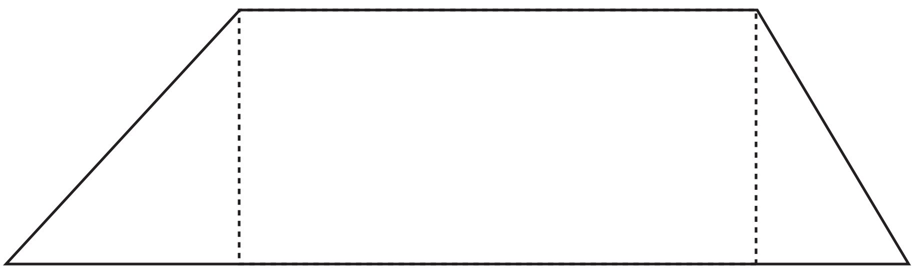
Parallelogram
Now that you know how to find the area of rectangles and triangles you can find the area of any polygon. You can imagine the shape as a collection of rectangles and triangles that you add or subtract to get the shape.

Think about a parallelogram:
- A parallelogram has a base and a height. Imagine a rectangle on top of the parallelogram.



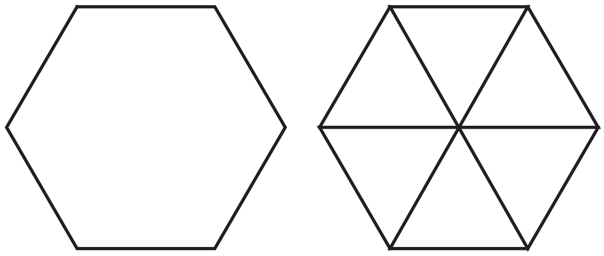
- Now you can see the parallelogram has a triangular part that sticks out from the rectangle. It also has a hole just the same size and shape on the other side. If you cut off the triangle on the right and paste it in the hole on the left you have a rectangle.
- This means that the area of a parallelogram is the same as the area of the rectangle formed by the base and the height.
- So the formula for the area of a parallelogram is A = base × height.

1. Trapezium

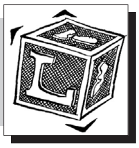


You can find the area of a trapezium if you see it as a rectangle and two triangles. You already know how to find the area of those.

2. Hexagon.

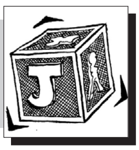


A regular hexagon is just a group of triangles. You know how to find the area of triangles so you can find the area of any regular hexagon.



What have you learned?

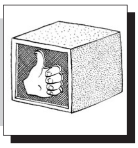
Perhaps you are a bit surprised to realize how simple it can be. You can see that any polygon can be divided into a number of triangles and rectangles. You can use your knowledge of the area of a rectangle, $A = l \times b$, and your knowledge of the area of a triangle, $A = \frac{1}{2}(b \times h)$, to find the area of any regular or irregular polygon.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about the perimeter of triangles and other polygons?
b. How do you think you will be able to improve your understanding of the area of triangles and polygons?
c. Write down one or two questions that you still have about the relationship between the area of a triangle and the area of a rectangle.
d. How will you use what you learned about triangles and polygons in your every day life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. find the perimeter of triangles and other regular and irregular polygons				
2. find the area of triangles and other regular and irregular polygons				
3. demonstrate that the area of a triangle is half the area of the rectangle that just fits around it.				

UNIT NINE

Measuring Circles

In this unit you will address the following:

Unit Standard 7450
SO1:
Apply relationships between common quantities in various contexts. (Mass and weight, distance and displacement, speed and velocity, volume and density, volume and surface area, area and perimeter, distance and time, volume and capacity.)

SO2:
Use measuring instruments to measure and calculate quantities in various contexts. (Quantities include all of: length, distance, mass, time, temperature, volumes of regular prisms, perimeter, area, weight, surface area, density, displacement and angles. Measuring instruments include all of: rulers, tape measures, scale, clocks, thermometers, capacity measuring instruments, and protractors.)

SO3:
Solve measurement problems in various contexts. (practical and non-practical processes, trigonometric right-angled heights and distances.)

Unit Standard 7464
SO1:
Identify geometric shapes and patterns in cultural products. (shapes of and decorations on cultural products such as drums, pots, mats, buildings, and necklaces.)

SO2:
Analyze similarities & differences in shapes & patterns, & effect of colour, used by cultures. (analyze similarities and differences in shapes and patterns, and the effect of colour, used by different cultures.)

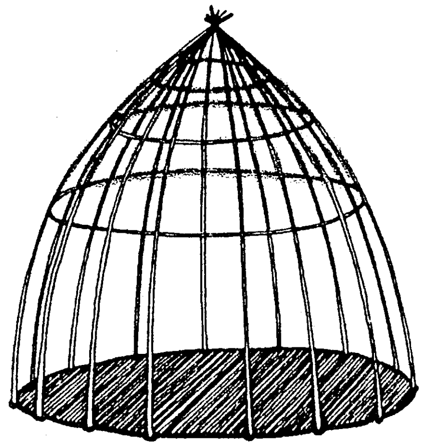
SO3:
Analyze and explain the way shapes and space are used in different epochs and cultures. (Architecture, town and settlement planning.)

To do this you will:
- explain how to derive the formula for calculating the circumference of a circle;
- use the formula to solve related problems;
- explain how to derive the formula to find the area of a circle;
- use the formula to solve related problems.

1. Circles long ago

For thousands of years people have built shelters for themselves using circular structures. They did this without having the kinds of measuring instruments we use today. Still they managed to create perfectly formed circles.

This example shows how Sotho builders constructed the circular frame for this beehive hut.



Time needed
15 minutes

Activity 1:
Circles long ago

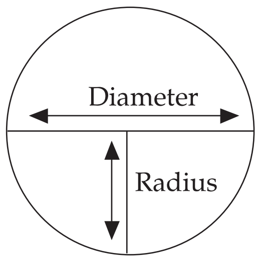
Work with a partner

Discuss the following:

1. What steps do you think the builders followed to make the frame?
2. They did not use rulers, measuring tapes or compasses. How do you think they measured the circle for the base of the hut, or the lengths for the circular hoops?
3. What methods, tools and computer programmes do you know of that would help people design buildings like these today?

2. Circumference, diameter and radius

- A straight line that joins to points on the edge of the circle and passes through its centre is called the diameter (d).
- A straight line from the centre of the circle to any point on the circle is the radius (r).
- The radius is half the length of the diameter.
- The distance all the way around the circle is the circumference (c).
- To calculate the circumference, you need to know something about the radius or diameter measurement.



Time needed
20 minutes

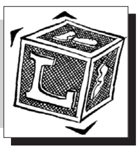
Activity 2:
Investigating Pi

Work with a partner

1. Find four circular objects of different sizes like the lid of a jar, a hoop or a cooldrink can. Trace around each one. Estimate and mark the centre of each circle.
2. Now take a piece of string and measure the circumference of each circle. Hold the string down at any point on the circumference. Carefully lay it around the circumference of the circle. It's easier if you do this together!
3. Now measure the length of the string on a ruler.
4. Use a ruler to measure the length the diameter.
5. Divide the length of the circumference by the length of the diameter. Your answer will be a little bit more than 3. This is called the ratio of the circumference to the diameter (circumference: diameter).
6. Use string and a ruler to measure the circumference and diameter of all your circles.
7. Record your measurements in a table like the one below. In the last column, record the ratio of the circumference to the diameter.

Name of circle	Circumference	Diameter	Circumference: Diameter
E.g. bowl			

8. Discuss what you notice about the ratio of the circumference to the diameter in all the circles you measured?



What have you learned?

Maybe you already knew how to measure the circumference using a piece of string. Now you have found out that when you divided the circumference length by the diameter length of each circle, your answer was just over 3 each time.

The ancient Greeks first discovered that the ratio between the circumference and the diameter in any circle is always just over 3 or about $3\frac{1}{7}$ or $\frac{22}{7}$.

The Greeks used their letter π pi to show this ratio. Mathematicians have been interested in this number for thousands of years. They have found that the number does not have an end and the numbers after the decimal point do not have a clear pattern. This means that π is an irrational number. Today we work with the value of π being equal to 3,14.

Now you know that circumference = $\underline{\quad \pi \quad}$
Diameter

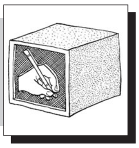
This means that you can also say that the
Circumference = $\pi \times$ diameter
Or
$c = \pi d$

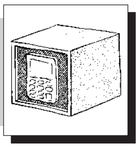
You also know that the diameter is $2 \times$ radius.
So another way to write $c = \pi d$ is $c = \pi \times 2r$
You can also write this as $c = \pi 2r$
or
$c = 2\pi r$

Time needed
20 minutes

Activity 3:
Using Pi to find the circumference of a circle

Add this activity to your portfolio. Show all your calculations.





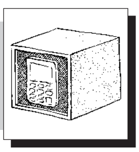
Work with a partner

Use the π key on your calculator or work with the value of π as 3.14 to do these tasks.

1. Use a large plate and a CD for this measurement task.
   a. Estimate what you think the circumference of each object is. Write down your estimate.
   b. Now measure the radius or diameter of each object.
   c. Use π on your calculator or work with the value of 3,14 to find the actual circumference. Remember $c = \pi \times 2r$.
   d. Check how close your estimate was.

2. Imagine a bicycle wheel turning. The distance a wheel travels when it turns around once is the length of its circumference. Try this with a plate. Make a mark at one point on the plat and then turn it like a wheel. Start and finish at the mark. Use this information to work out the following problem:
   a. The spokes of a bicycle wheel are 45cm long and the outer rim is 6,5cm thick. How far does the wheel travel when it turns around once?
   b. Without using your calculator, fill in the table below to show how far the wheel will travel if it turns 10 times, 100 times, 5 times etc. Think of quick ways to do this. Round your measurements to the second decimal place.

No of turns	**1**	**10**	**100**	**5**	**50**	**100**	**200**	**500**	**1000**	**6000**	**10 000**
Distance Travelled	?										
Calculator Answer											

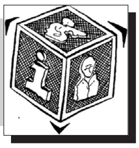


   c. Do the same calculations again with your calculator. Write these answers in the bottom row.

Work with a partner

d. Share your findings with a partner. Discuss and compare some of the quick ways you used to find the missing answers. Compare each of your calculator answers.

3. The distance from the Northern border in South Africa in the Limpopo Province to Cape Town in the Western Cape is 1715km. So imagine a distance of 40 000km, across the earth's circumference! The equator measures 40 000km across the centre of the earth. Use this measurement to find the circumference of the earth.



Time needed
30 minutes

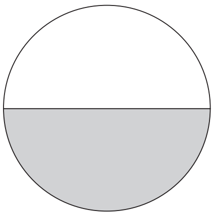
3. Area of a circle

You use the value of π to find the area of a circle as well. Let's see how this works.

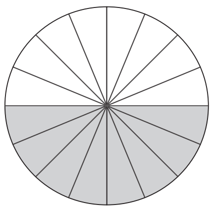
Activity 4:
Finding the area of a circle

Work in pairs

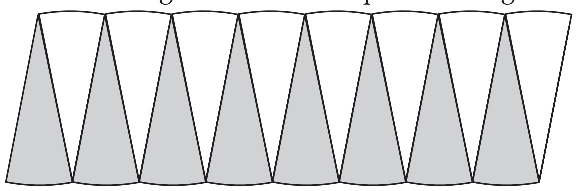
1. Draw a circle like this and shade half of it.



2. Now divide it into 16 equal segments



3. Cut out the segments and then put them together like this



a. What shape have you made?
b. Discuss the length of the rectangle in relation to the circumference of the circle. Can you write this down?
c. Think about the breadth of the rectangle in relation to the radius of the circle. Can you write this down?
d. Try and write the formula for the area of the rectangle using π and r.
e. What is the formula for the area of a circle?

What have you learned?

Perhaps you already know the formula for measuring the area of a circle but maybe you do not know why. In this activity you have seen the explanation in a simple and clear way. Here is a summary of that process:

- The shape you made is almost a rectangle.
- The length of the rectangle is $\frac{1}{2}$ the circumference of the circle. Another way to write this is

$\frac{1}{2} \times 2\pi r$

$= \pi r$

- The breadth of the rectangle is equal to the radius of the circle or r.
- Remember that the area of a rectangle = l × b.

In this case you will write it as

$\pi r \times r = \pi r^2$

- So the formula for the area of a circle is πr^2 .

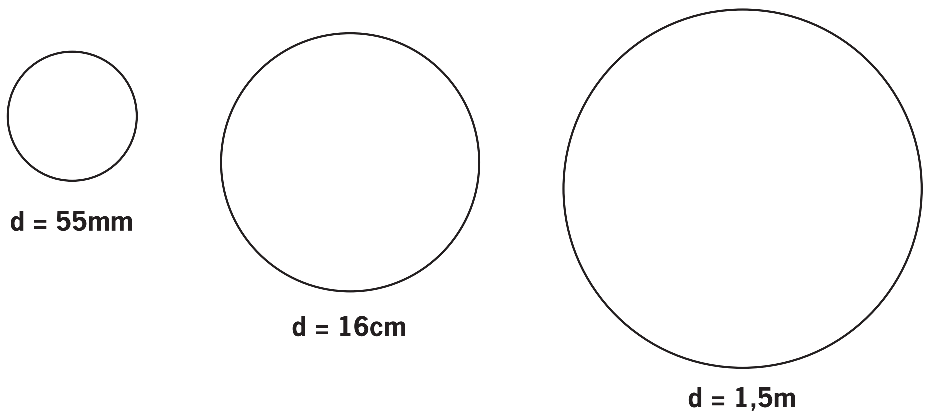
Activity 5: Finding the area of a circle

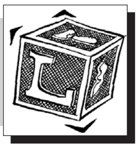
Time needed 30 minutes

Work in pairs

Find the areas of the following circles. Remember to give your answers in square units (mm^2, cm^2 or m^2):

a) The lid of a bottle
b) A dinner plate
c) A circular area in a garden





What have you learned?

To work out the area of these circles you used the formula πr^2. The diametre of the first circle is 55mm. This means the radius is 27,5mm. So the calculation for the area of the first circle will look like this:

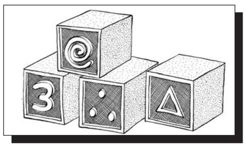
$A = 3{,}14 \times 27{,}5 \times 27{,}5$

$= 2374{,}625mm^2$

You will work out the areas of the other circles using the same formulae.

This should all make more sense to you now that you know how the formula was worked out to begin with.

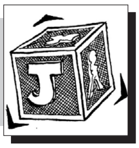
Give yourself lots of practice working out the areas of other circles in the world around you so that you feel confident.



Linking your learning with your ECD work

Ideas for exploring circles with young learners

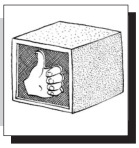
- Children can sort circles from biggest to smallest, smallest to biggest. Encourage them to talk about the circles and describe their properties using their own language like curved, round, wide etc. If you teach a multilingual class, encourage the children to use the same words in different languages.
- Children can compare half circles and circles and describe them. For example, the half circle has one straight side and one curved side. The circle has one side that goes all the way around.
- Children can match square and rounds lids to their correct tins or bottles.
- Children can trace around the ends of the cylinders or the lids and compare the different sizes of the circles they drew and then match these with the lids or cylinders they belong to.
- They can also use the ends of the cylinder to print with, by dipping them in a thick paint and starch or a mud and starch mixture.
- Older children can see how many circles they can fit on a page without them overlapping. Talk about the gaps that the pattern makes.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about the area and circumference of a circle?
b. How do you think you will be able to improve your understanding of the formulae for calculating are and circumference of a circle?
c. Write down one or two questions that you still have about area of circles.
d. Write down one or two questions that you still have about circumference of circles.
e. How will you use what you learned about circles in your every day life and work?

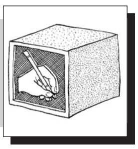


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Explain how to derive the formula for calculating the circumference of a circle				
2. Use the formula to solve related problems				
3. Explain how to derive the formula to find the area of a circle				
4. Use the formula to solve related problems				



Assignment 5:

The earth is far from being the largest planet. Do some of your own research to find out how big some of the other planets are. If you only find their diameter measurement you now know how to work out the circumference!

You can also find out the diameter measurement from the circumference by changing the formula around and instead of calculating $C = \pi D$, work out D as $C\pi$

Record your answers in a table. Show any calculations that you had to do to find any of the measurements that were not given in your source.

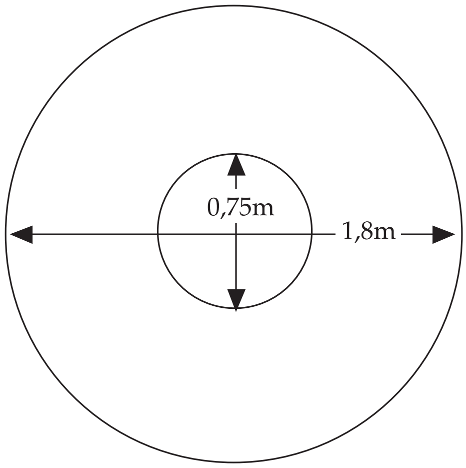
For your own reference, and to share with your colleagues, provide information in the last column about the source of your information e.g. name of book; internet site.

Compare and discuss your research findings with your colleagues.

Assignment 6:

Phindi is a community nurse. She wants to build a herb garden in the grounds of the clinic. She wants it to look like the sketch below, with one larger circle surrounding a smaller circle. She wants to fence each of the circular borders.

The radius of the inner circle is 0,75m
The diameter of the outer circle is 1,8m



a. How much fencing will Phindi need for each border?
b. The fencing normally costs R59 per metre. On sale it is 15% less. How much will she pay for the fencing at the sale price? (round your answer to the second decimal place).

UNIT TEN

Investigating surface area of Prisms and Cylinders

In this unit you will address the following:

Unit Standard 7450

SO1:

Apply relationships between common quantities in various contexts. (Mass and weight, distance and displacement, speed and velocity, volume and density, volume and surface area, area and perimeter, distance and time, volume and capacity.)

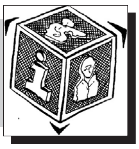
SO2:

Use measuring instruments to measure and calculate quantities in various contexts. (Quantities include all of: length, distance, mass, time, temperature, volumes of regular prisms, perimeter, area, weight, surface area, density, displacement and angles. Measuring instruments include all of: rulers, tape measures, scale, clocks, thermometers, capacity measuring instruments, and protractors.)

SO3:

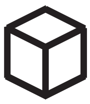
Solve measurement problems in various contexts. (practical and non-practical processes, trigonometric right-angled heights and distances.)

To do this you will:
- explain what surface area is;
- investigate patterns and relationships between the number of cubes that join together and their surface area measurements;
- solve problems relating to surface area of cubes and cuboids;
- use what you know about finding the area of triangles and the Pythagoras theorem to find the surface area of prisms with a triangular or hexagonal base;
- use the formula you learnt for finding the area of a circle to calculate the surface area of a cylinder.



1. Surface Area

So far you have investigated areas of flat 2-D shapes only. But if you want to calculate the area of all the faces of a 3-D object, you need to find the surface area of that object. Let's investigate what this means.



- This cube has 6 faces. You can only see three of the faces.
- Each face measures 1cm by 1cm. So we know that each face has an area of $1cm^2$.
- When you add the area of all the faces of the cube you get $6cm^2$.
- So the surface area of a cube or of any prism is the sum of the area of all its faces.

Time needed
30 minutes

Activity 1:
What is Surface Area?

Work with a partner

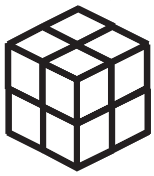
1. Find the total surface area of this cube. Remember to find the area of each face first.
2. How many small cubes will you need to make a 3 × 3 cube? What is the surface area of a 3 × 3 cube?
3. Do a calculation to find the size of cube that has
   a. A surface area of 150cm^2
   b. A surface area of 864cm^2
   c. Discuss and share your methods and answers with a partner.

Time needed
30 minutes

Activity 2:
Surface area of rectangular prisms

Work with a partner

You found that the surface area of one of these cubes was 24cm^2. So two cubes have a surface area of 48cm^2. Predict the surface area of the two cubes put together with one side joining.



1. Find a pattern in the relationship between the numbers in each column. Use what you find out to fill in the missing numbers in the table.

1 cube	24		
2 cubes	48	2 cubes together	40
3 cubes	72	3 cubes together	56
4 cubes	96	4 cubes together	72
5 cubes	120	5 cubes together	88
6 cubes		6 cubes together	
7 cubes		7 cubes together	
8 cubes		8 cubes together	
9 cubes		9 cubes together	
10 cubes		10 cubes together	

2. Can you see another pattern when you compare the numbers in the second column with the numbers in the last column?

What have you learned?

You have learned that the surface area of a polyhedron is the sum of the areas of its faces. In a cube all the faces are congruent. There are six faces in a cube. So the surface area is 6 times the area of one face. You investigated patterns in collections of cubes that have the same surface area You found that when you join cubes 2×2 cubes together, the surface area increases by only 16 units because two sides, each with a surface area of 8 units are lost each time you add another cube. But when you keep adding 2×2 cubes separately to a stack of cubes, where the sides do not join, and then count their total surface area, the surface area increases by 24 units each time.

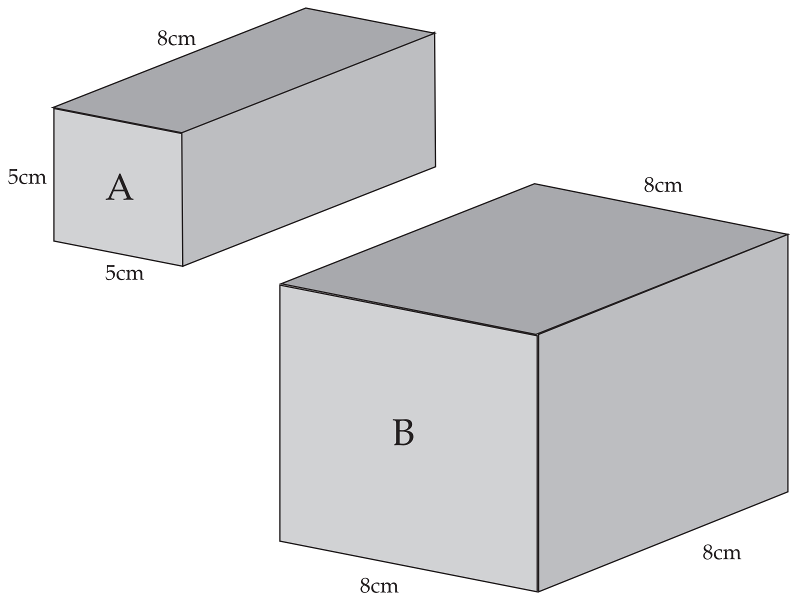
Activity 3:
Solving problems with surface area

Time needed
40 minutes

Do this activity on a separate paper and add it to your portfolio

Work alone

At a toy factory in a job creation project in the Free State Province, members of the project make hollow blocks out of sheets of plastic material. These are then sold to ECD centres all over the country. The plastic material comes in sheets. It is cut to the required lengths to make these two shapes of blocks.



1. Calculate the surface area of block A and B to find out how much plastic you need to make each block.
2. Use your calculation to find the cost of (i) 10 (ii) 100 (iii) 1 000 of each block
3. Show your findings in a table.
4. Compare your results with a partner's.

5. Use the measurements to draw a net on paper or cardboard of each shape. Cut and fold the net to see if it folds neatly. If not try again until your measurements are accurate.

What have you learned?

You worked out that the surface area of block A was 210cm^2 and you multiplied by multiples of 10 to find the cost of more.

You practiced measuring accurately so that you could design nets for each shape. You realized that measuring accurately is important for calculating surface area and designing three dimensional nets.

Linking your learning to your work with children

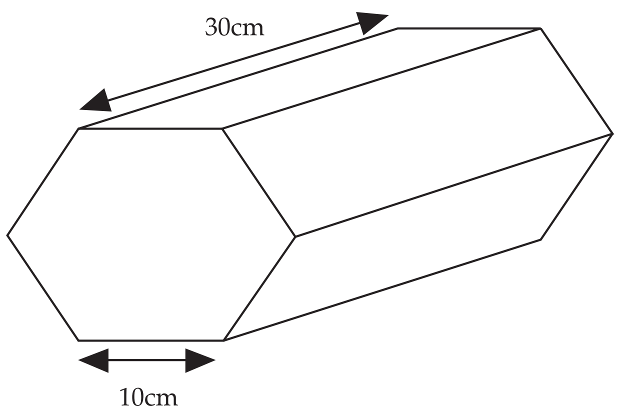
- Children work in pairs or small groups to build models by sticking cardboard boxes together in different ways. They can use boxes like cereal boxes, tea boxes etc.
- Let them cover their boxes with paper. They can then find out and talk about which models need more and less paper.

2. Surface Area of Prisms

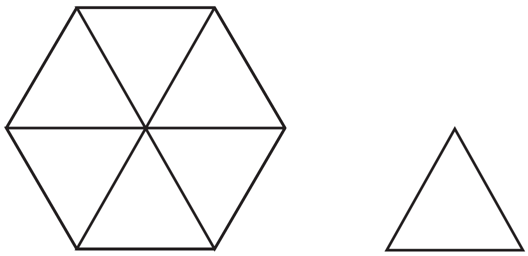
Read through this section slowly together with a partner. Make sure you understand the explanation of each step of the process. Ask your trainer for help if there is anything you do not understand.

To find the surface area of a prism you need to know the area of the two parallel bases, and the areas of each of the parallelogram faces. Take the example of a right hexagonal prism. These are the steps we follow:

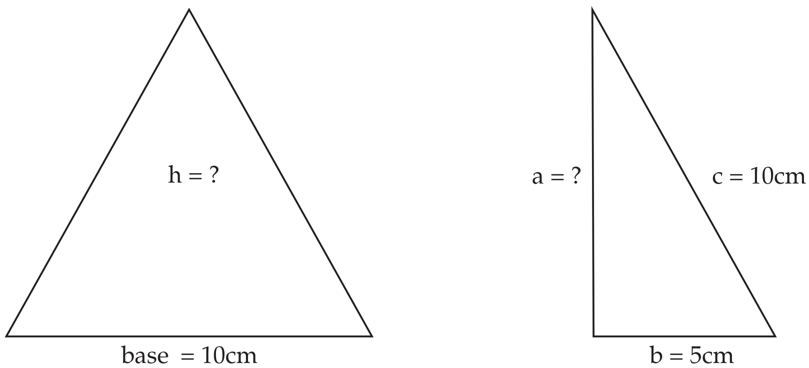
The sides of the regular hexagon are 10cm and the length of the prism is 30cm.



1. First we need to find the area of the bases or end faces.
    a. Remember that we find the areas of polygons by dividing them into triangles and rectangles. This hexagon is made up of six equilateral triangles, each 10cm on a side. We can find the area of one triangle and multiply by 6 to get the area of the hexagon.



    b. You know that the area of the triangle is A = $\frac{1}{2}$ base × height. The base is 10cm. But you don't know the height! Let's use the Pythagorean Theorem to find out.



Half of the equilateral triangle forms a right angle with a hypotenuse of 10cm and one length = 5cm. We now want to find the length of a.

$a^2 + b^2 = c^2$
$a^2 = c^2 - b^2$
$a^2 = (10cm)^2 - (5cm)^2$
$a^2 = 100cm^2 - 75cm^2$
$a^2 = 75cm^2$

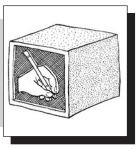
We can use the √ key on the calculator to find the square root of a:
a = 8.66cm
The height of the right triangle = height of the equilateral triangle.

So, the area of the equilateral triangle is
Area = $\frac{1}{2}$ (base × height)

= $\frac{1}{2}$ (10cm × 8.66cm)
= 43.3 cm^2

c. The area of the hexagonal base is A = 6 × 43.3cm^2 = 259.8cm^2
d. There are two hexagonal bases. The area of the bases = 2 × 43.3cm^2 = 86.2cm^2
e. There are six sides, each a rectangle of 10cm x 30cm. The area of each face is 10cm × 30cm = 300cm^2 . The total area of the faces is 6 × 300cm^2 = 1 800cm^2
f. The surface area of the hexagonal prism is the area of the hexagonal bases plus the area of the rectangular sides = 86.2cm^2 + 1 800cm^2 = 1 886.2cm^2

Time needed
15 minutes

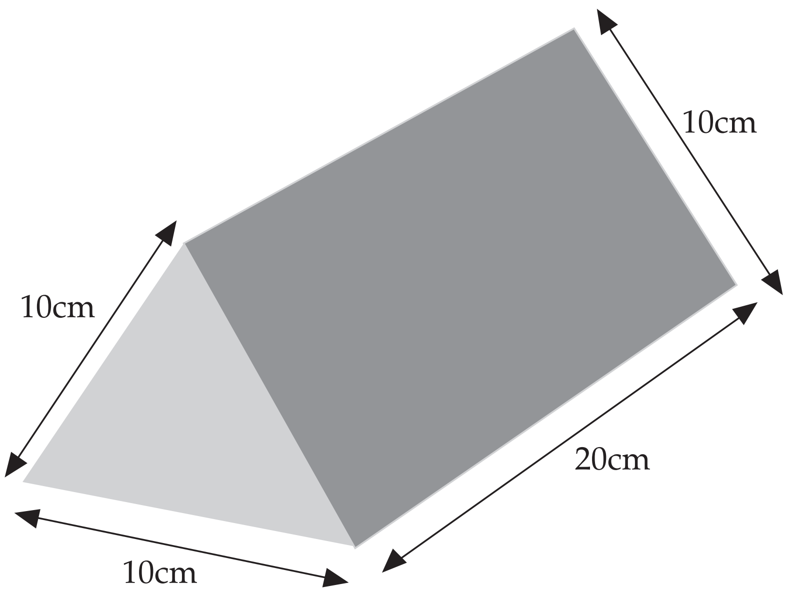


Activity 4:
Finding the surface area of a triangular prism

Do this activity on a separate paper and add it to your portfolio

Work alone

1. Use the steps above to find the surface area of this triangular prism.
   Here are some clues:
   - The bases are equilateral triangles with sides of 10cm. In the example we calculated the area of an equilateral triangle with sides of 10cm. Use this result for the area of the bases.
   - The other faces of the prism are rectangles 20cm long and 10cm wide.



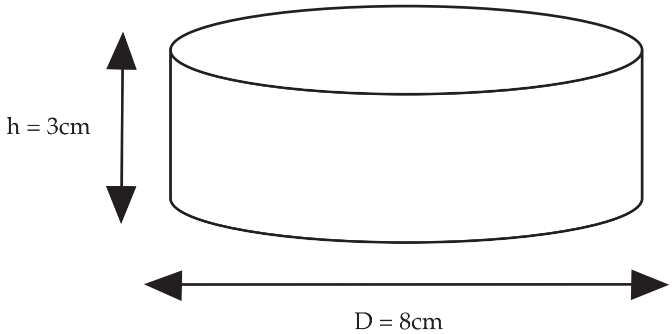
2. Check your solutions with a partner. If you don’t have the same answers re-do your calculations to find if you made mistake.

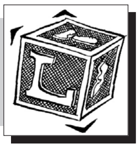
Time needed
15 minutes

Activity 5:
Finding the surface area of a cylinder

Work alone

1. Find the total surface area of a closed cylindrical tin with the dimensions given.
   Hint: Draw the net for the cylinder showing top, bottom and sides. Find the area of each shape.

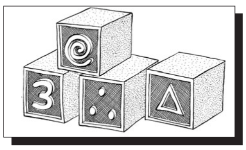




What have you learned?

You have learned that the surface area of any polyhedron is the sum of the surfaces of its faces. Even when you don't know a formula for the area of a given shape, you can find the area by dividing the shape into rectangles and triangles.

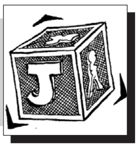
- For prisms, the surface area is the sum of the areas of the two parallel bases and the parallelogram sides.
- For cylinders your net will show you that the sides will be a rectangle. One dimension is the height of the cylinder, and the other the same is the circumference of the cylinder. For closed cylinders you add the area of the circular tops and bottoms.



Linking your learning with your ECD work

Investigating prisms and cylinders

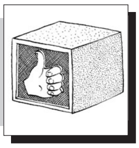
- Children can find examples of different cylinders and prisms used in packaging.
- They can undo the boxes and talk about the different shapes that make up each container.
- They can paint the faces that are the same shape in the same colour.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about surface?
b. How do you think you will be able to improve your understanding of surface area?
c. Write down one or two questions that you still have about surface area of cuboids.
d. Write down one or two questions that you still have about prisms and cylinders.
e. How will you use what you learned about surface area in your every day life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Explain what surface area is				
2. Investigate patterns and relationships between the number of cubes that join together and their surface area measurements				
3. Solve problems relating to surface area of cubes and cuboids				
4. Use what I know about finding the area of triangles and the Pythagoras theorem to find the surface area of prisms with a triangular or hexagonal base.				
5. Use the formula I learnt for finding the area of a circle to calculate the surface area of a cylinder				

UNIT ELEVEN

Investigating Volume

In this unit you will address the following:

Unit Standard 7450
SO1:
Apply relationships between common quantities in various contexts. (Mass and weight, distance and displacement, speed and velocity, volume and density, volume and surface area, area and perimeter, distance and time, volume and capacity.)

SO2:
Use measuring instruments to measure and calculate quantities in various contexts. (Quantities include all of: length, distance, mass, time, temperature, volumes of regular prisms, perimeter, area, weight, surface area, density, displacement and angles. Measuring instruments include all of: rulers, tape measures, scale, clocks, thermometers, capacity measuring instruments, and protractors.)

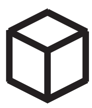
SO3:
Solve measurement problems in various contexts. (practical and non-practical processes, trigonometric right-angled heights and distances.)

Unit Standard 7463
SO2:
Illustrate changes in size & shape of appearance of objects as result of changes in orientation.

To do this you will:
- derive and use the formula for calculating the volume of cubes and cuboids;
- express volume in cubic units of length and capacity;
- investigate measurements of cubes and cuboids with the same volume but different surface areas;
- use the formula for finding the area of triangles and rectangles to find the volume of hexagonal and triangular prisms;
- use the formula to find the area of a circle to find the volume of a cylinder;
- use this range of skills and knowledge to solve real life problems.

1. Volume

Volume means the amount of space that something takes up.



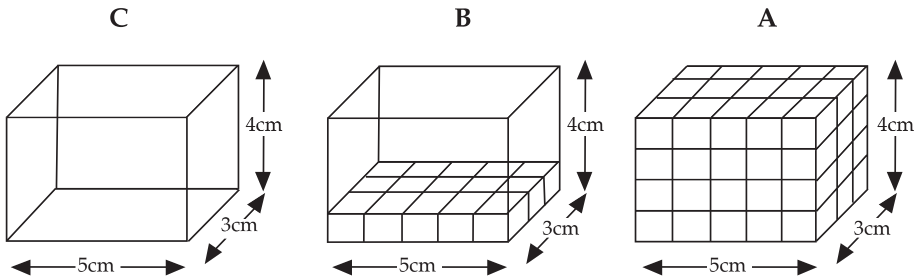
This cube has a length of 1cm, a height of 1cm, a width of 1cm. So it measures 1cm × 1cm × 1cm. We say that its volume is 1 cubic centimetre, written as $1cm^3$.

Time needed
30 minutes

Activity 1:
Investigating volume

Work alone

1. Use these sketches to find an easy method to calculate the volume of the cube.
2. Discuss and compare your methods with your partner.



What have you learned?

You probably found that the quickest way to calculate the volume (number of $1 \times cm^3$ cubes in the cuboid) was as follows:

- Count the number of cubes in the length × the number of cubes in the breadth × the number of cubes in the height. This would be written like this: $V = l \times b \times h$
- In the case of the cube above this is $V = 9 \times 9 \times 9cm^3 = 729cm^3$. So the formula for volume is $V = l \times b \times h$.

If you know the volume and any two of the dimensions, you can calculate the third measurement. For example: a cube has a volume of $27cm^3$. The height is 3cm and the breadth is 3cm.

You can calculate $3 \times 3 = 9$, divide 27cm by 9, and you find the length of the cube: 3cm.

Time needed
30 minutes

Activity 2:
Missing Measurements

Work alone

1. Use the above method to calculate the missing measurement in each case below.

Volume (mm^3)	900	600	540	875
Length (mm)	15	10		5
Width (mm)	4		12	25
Height (mm)		30	9	

2. Rewrite the table above in measurements of cm^3. Remember there are 10mm in each cm. Check your answers and methods with a partner's.

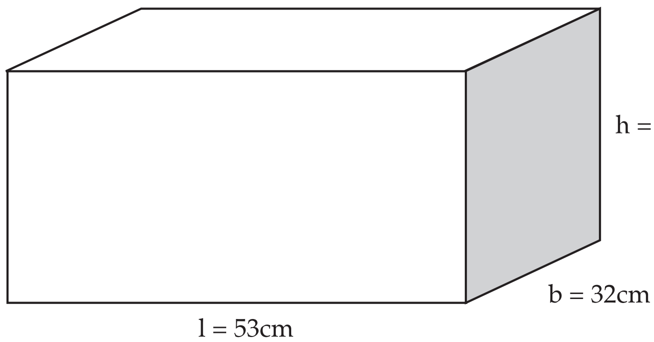
Time needed
30 minutes

Activity 3:
Packing Boxes

Do this activity on a separate paper and add it to your portfolio

Work alone

The fathers at Bantwana Bami make wooden blocks to sell. They pack the blocks in cardboard boxes that look like the one below. Each container measures 40cm × 32cm × 53cm.



1. What is the volume of the box?
2. They pack 10 of these boxes in a larger cardboard container. What will the volume of 10 boxes be?
3. Draw two sketches of different containers you could make to hold 10 such boxes.
4. Compare your designs with a partner. Use a calculator to check your calculations.
5. Estimate which container design uses less cardboard. Do a calculation to find out if you were right.
6. The fathers also sell smaller boxes of $1cm^3$ blocks. Normally they pack 50 in a box, but they are offering a special deal. If you buy 50, you get 10 more for free. They are expecting lots of orders, so they make up boxes with 60 blocks in all. How many different ways can you find to do this? All the boxes must have the same volume of $60cm^3$. Write down the ways. Use drawings to show the measurements of the containers you would use.
7. Which design has the smallest surface area? Which has the largest?

Do this activity on a separate paper and add it to your portfolio

What have you learned?

To calculate the volume of a cube or cuboid you use the formula l × b × h.

Two cuboids with the same volume can have different surface areas. This may be useful to know when you have to pack a number of boxes into a larger container like a crate or the back of a truck.

Maybe you were able to do some of these calculations mentally (in your head), using your knowledge of multiplication. Some calculations were easier when you used a calculator.

You gave some of your answers in cubic measurements like mm3 . When you converted some of these to cm you divided by 10 because you know that there are 10 mm in one cm.

Linking your learning with your ECD work

- Children can try different ways of packing blocks or other boxes into larger containers or in piles. Encourage them to talk about and compare and justify their different ways. A child may say something like "My pile is three stacks wide and 4 stacks across!" or "Mine is wider than yours!" and "Yes, but mine is taller!"

2. Units of Volume and Capacity

The standard unit of volume in the metric system is the litre (abbreviated ℓ). One litre is equal to 1000 cubic centimetres in volume. Other units of volume and their equivalents in litres are as follows:

1 millilitre (mℓ) = 0.001 litre = 1 cm^3
1 centilitre (cℓ) = 0.01 litre = 10 cm^3
1 decilitre (dℓ) = 0.1 litre = 100 cm^3
1 kilolitre (kℓ) = 1000 litres = 1 m^3

Activity 4:
Converting units

Time needed
15 minutes

Work alone

1. Name some quantities that are measured in
   a. millilitres
   b. litres
   c. kilolitres

2. Express these units in litres
   (i) 345mℓ (ii) 2,5kℓ (iii) 45dℓ (iv) 4567mℓ (v) 43,3kℓ

3. How many millilitres in
   (i) 4,56ℓ (ii) 56,1ℓ (iii) 45dℓ (iv) 1kℓ

4. Check your solutions with a partner. You can use a calculator to see if you were correct e.g. 2,5kℓ × 1000 = 2 500ℓ

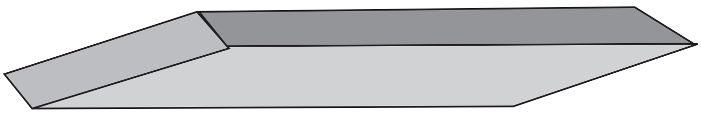
3. Volume of Prisms

Read through the following text carefully.

A rectangular or square box is one kind of prism. Now let's explore the volume of different shaped prisms.

You remember that the volume of a cuboid, or rectangular box, is $V = l \times w \times h$. Since $l \times w = A$, the area of the base, we could also say that the volume is the area of the base time the height.

How could we go about finding the volume of something that looked like this?

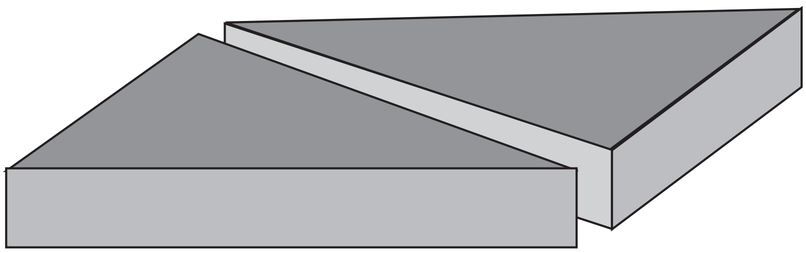


A slanted box like this is called an oblique

It is easy to find the answer if we use our imagination. Think about a stack of paper – let's say a ream of blue A4, stacked up neatly the way it arrives in the package.

Now, let's gently push the stack so that it looks like the figure above. What can you say about the volume? It's the same ream of paper, stacked just as tightly, so the volume is the same as it was before, though the stack has a different shape. The volume is equal to the area times the height.

Now if we straighten that stack again, we can do another thought experiment. This time, let's imagine we can cut the ream of paper neatly in two.



Now we have two triangular prisms. What is the volume of each? Clearly each prism has a volume of half of the volume of the ream. The ream's volume is $V = l \times w \times h$,

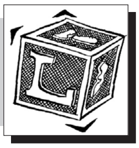
or V= the area of the ream × height.

That means each triangular prism has a volume of $\frac{1}{2}$ of the area of the ream times the height.

V = $\frac{1}{2}$ length × width time height.

But what is the area of the triangle? It is $\frac{1}{2}$ length × width

Can you see that the area of the base of each triangular prism is $\frac{1}{2}$ the area of the ream. So for the triangular prism we can also say the volume is the area of the base (in this case A= $\frac{1}{2}$ × l × w) times the height!

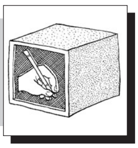


You found out earlier that you could use your knowledge of the area of rectangles and triangles to find the area of any polygon. Similarly, you can look at any prism as a combination of rectangular prisms (cuboids) and triangular prisms. With this knowledge you can find the volume of any prism.

Time needed
20 minutes

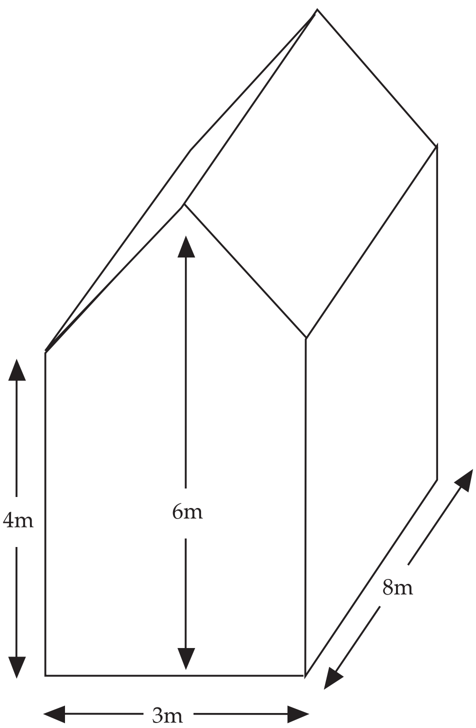
Activity 5:
Find the volume of a storage room

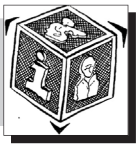
Do this activity on a separate paper and add it to your portfolio



Work alone

1. Find the volume of this storage room. It is a right pentagonal prism. The faces are the front and back of the room. Find the area of the faces and multiply by the distance between them. In this case it's the length of the room, rather than the height.
2. If the whole storage room is to be painted, including the roof, what is the surface area to be covered?



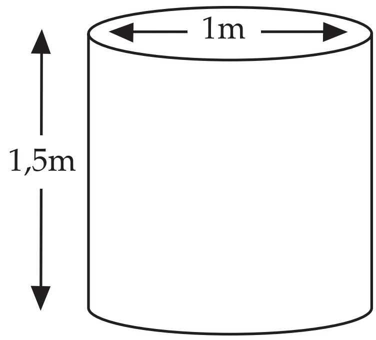


4. Volume of Cylinders

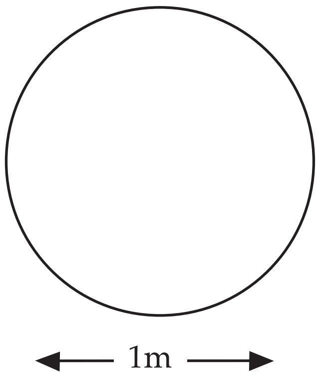
Read the information together with a partner. Make sure you understand each step.
If you have difficulty, ask for help from another colleague or from your trainer.

With volume, cylinders are just like prisms. All we need to know is the area of the base and the height of the cylinder.

1. Find the area of its base and multiply by the height. Think about the capacity of a water tank that is 1 metre in diameter and 1,5 metres tall.

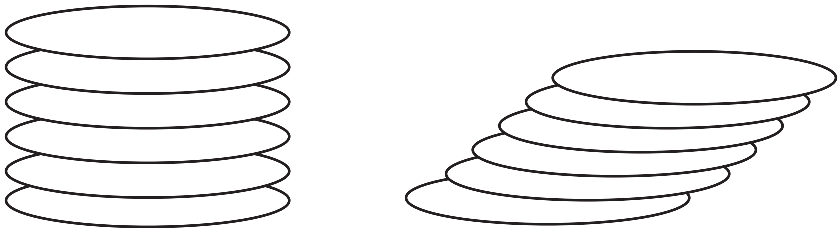


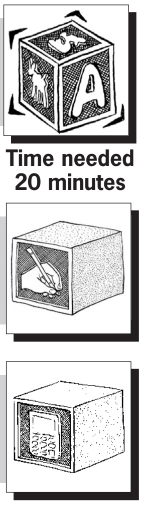
2. First we'll find the area of the circular base. Remember that the area of a circle is $A = \pi r^2$. The radius is $\frac{1}{2}$ the diameter, or 0,5 metres. Let's use 3,14 for π . Then
   $A = 3{,}14 \times (0{,}5m)^2$
   $= 3{,}14 \times 0{,}25m^2$
   $= 0{,}7850m^2$



3. Now we multiply the area of the circle by the height of the tank to find the volume.
   $V = A \times h$
   $= 0{,}\ 7850m^2 \times 1.5m$
   $= 1{,}\ 1775m^3$

It is the same with oblique cylinders. Imagine a stack of biscuits. If we slide the stack so that it tilts a bit we haven't changed the volume of the stack.





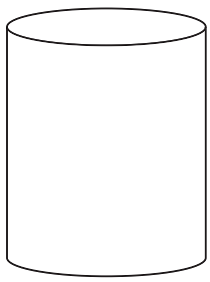
Time needed
20 minutes

Activity 6:
Finding the Volume of cylinders.

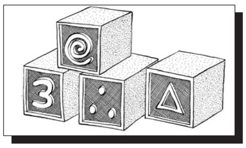
Do this activity on a separate paper and add it to your portfolio

Work alone

1. Hot chocolate is sold in cylindrical tins like this. The diameter of the tin is 8cm. The tin is 15cm high.



2. What is the volume of the tin?
3. The manufacturers decide to change to rectangular tins. The new tin must have the same volume as the circular tin. Suggest some possible measurements for the rectangular tin.
4. Compare your answers and calculations with a partner's.



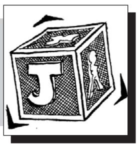
Linking your learning with your ECD work

Exploring Volume

- Let children explore and talk about different shaped containers eg. "This tin has two circles at the bottom and one curvy part that goes all the way around."
- Children estimate which container takes up more room in a box. They test to see if their predictions were right.

Exploring Capacity

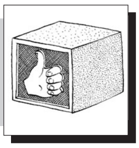
- Children can guess which containers will hold a litre, more than a litre or less than a litre.
- They can pour from a marked litre container filled with water or sand, into other containers, one by one, to test their predictions.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about calculating volume?
b. How do you think you will be able to improve your understanding of volume and surface area?
c. Write down one or two questions that you still have about the volume of prisms and cylinders.
d. How will you use what you learned about volume in your every day life and work?

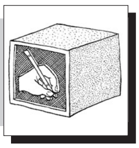


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Derive and use the formula for calculating the volume of cubes and cuboids.				
2. Express volume in cubic units of length and capacity				
3. Investigate measurements of cubes and cuboids with the same volume but different surface areas.				
4. Use the formula for finding the area of triangles and rectangles to find the volume of hexagonal and triangular prisms.				
5. Use the formula to find the area of a circle to find the volume of a cylinder.				
6. Use this range of skills and knowledge to solve real life problems.				



Assignment 7: Build a one litre cuboid

1. Work out several ways you could make a box (cuboid) with a capacity of 1 000cm^3.
2. Calculate the surface area of each box. Find which box gives you the smallest surface area.
3. Use grid paper to make a net and build your one litre cuboid.

UNIT TWELVE

Views and Maps

In this unit you will address the following:

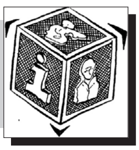
Unit Standard 7461
SO1:
Read, interpret and use maps, to depict and make sense of real locations, distances and position (Street maps: local and national maps.)

SO2:
Draw maps according to scale. (Non-contoured maps.)

Unit Standard 7463
SO1:
Describe and represent the position and change in position of an object in space. (words, rough sketches and abstract representation on a Cartesian plane.)

SO2:
Illustrate changes in size & shape of appearance of objects as result of changes in orientation.

To do this you will:
- name the different views of the same object, seen from different positions;
- draw different views of the same object seen from different viewing positions;
- draw different views of block "buildings" on a dotted grid;
- use numbers to describe the floor plan for different block "buildings";
- use a given floor plan, represented by numbers, to draw or build different block "buildings";
- interpret maps drawn on coded grids and that use a scale; answer questions relating to the information shown;
- draw your own map of a familiar landmarks in your community using the conventions of a coded grid and choosing a scale appropriate to the distances you want to show .



1. Different views

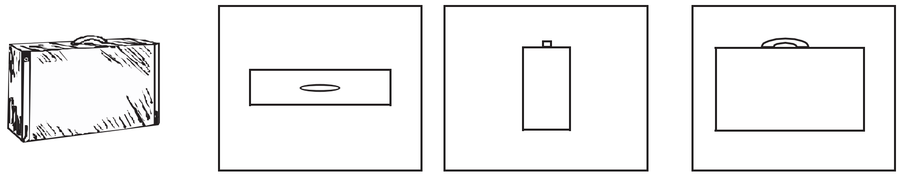
There are different ways to represent 3-dimensional objects in 2-dimensions in the form of drawings or plans. There are certain conventions that we follow and certain terms that we use. It is important to know what these conventions and terms are so that we all understand visual representations of objects in the same way. Conventions like these are part of social knowledge. We do not make up the rules as we go along, but rather follow methods that everyone uses and understands.

Activityy 1:
Naming different views

Time needed
10 minutes

Work with a partner

Vuyi bought a camera to take photographs at work. She first practiced using her camera by taking "close up" photos of this case.

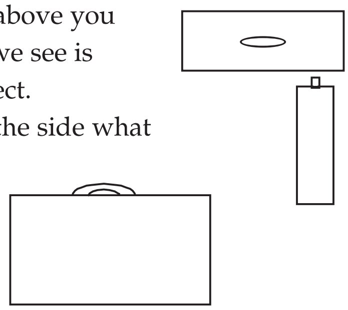


1. Where do you think Vuyi was standing when she took each of these photographs?

What have you learned?

You probably know from your own experience that an object looks different from different positions.

- When you look at an object from directly above you see the aerial view. The drawing of what we see is called the plan or the floor plan of the object.
- When you look at an object directly from the side what you see is called the side view.
- When you look at an object directly from the front what you see is called the front view.



Activity 2:
Take your own photos

Time needed
20 minutes

DICTIONARY:

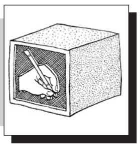
Aerial view –

Work alone

Choose an object such as a cereal box or cold drink can for this activity. Pretend you have a camera and you are taking photos of the object you have chosen.

1. Draw what you see when you take a "photo" of the object from directly above (aerial view).
2. Draw what you see when you take a "photo" of the object directly from the front (front view).
3. Draw what you see when you take a "photo" of the object directly from the side (side view).
4. What happens to the object in your photos when you move closer to the object?
5. What happens to your photos when you move further away from the object?

6. Now move to two different places and draw what your photo looks like from each of these points.
7. Discuss and review your drawings with a colleague.



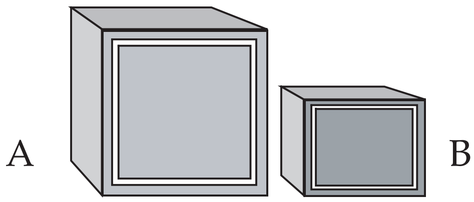
Activity 3:
Different views of a box

Do this activity on a separate paper and add it to your portfolio

Work alone

Look at the drawings of these two boxes.

1. Draw the aerial view of the two boxes together.
2. Draw the side view of the two boxes (from point A on the left).
3. Draw the side view of the two boxes (from point B on the right).
4. What will you draw if you look at the two boxes from directly behind?
5. Discuss and compare your drawings with a partner. Re-draw any of the views that you think you drew incorrectly.



What have you learned?

Before doing this activity perhaps you knew the different terms we use to describe different views of the same object. But maybe you have never drawn these different views before. It may have forced you to think at first but with practice you have gained confidence.

Choose a few more objects you can practice drawing from different viewpoints and then name the different views you have drawn each time. When you observe pictures of photographs in books and newspapers think about what views these pictures show. Try and visualise the same images from different viewing points.

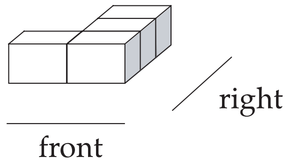
Time needed
20 minutes

Activity 4:
Block Buildings 1

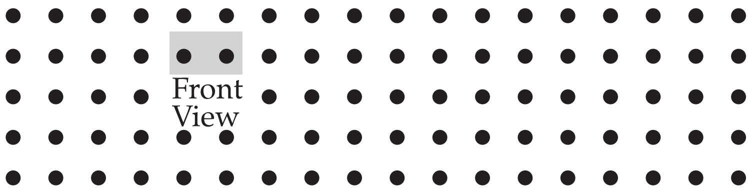
Work with a partner

You can represent drawings of block buildings in a plan. In this activity you will view the object “straight on”, not from above or from the side.

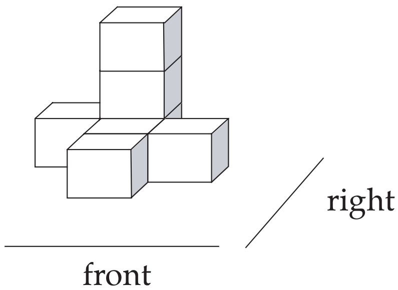
1. Look at this building of blocks. On the dotty paper below, draw each of the following:
    a. The front view of the building
    b. The aerial view of the building
    c. The side view (from the right) of the building
    d. The side view (from the left) of the building
    e. The view from directly behind

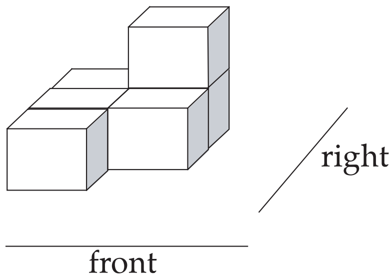


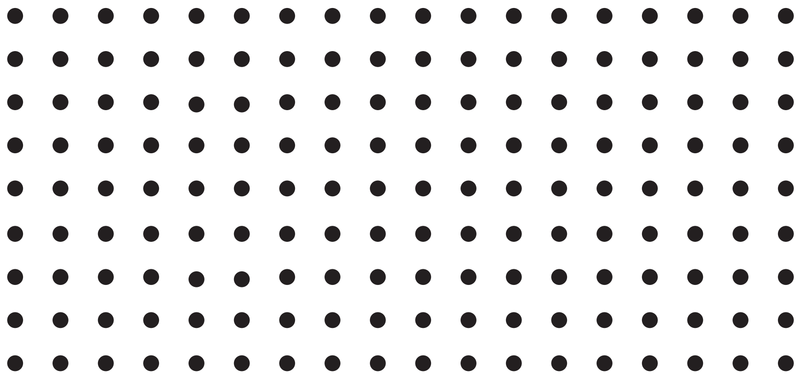
The first one has been done for you:



2. Now do the same for the two block buildings below.

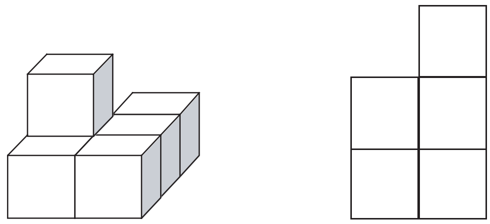






Floor plans

When you view an object from above it is called an aerial view. A plan of this view is called a floor plan.



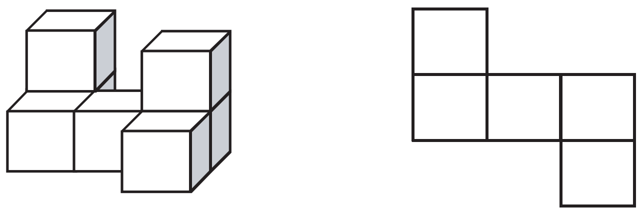
You can use numbers to show how many blocks there are in each stack, like this:

	1
2	1
1	1

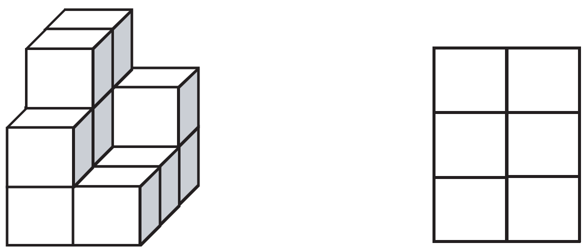
Activity 5:
Block Buildings 2

1. In each case below you can see a sketch of a building and its floor plan. Fill in the numbers on the floor plan to show how many blocks in each stack. Say how many blocks are there in each building.

a)

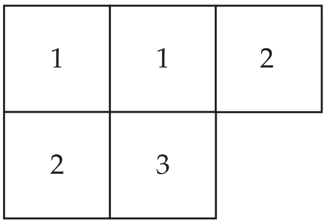


b)



2. Use cubes or do a drawing to build a block building for each of the following floor plans:

a)



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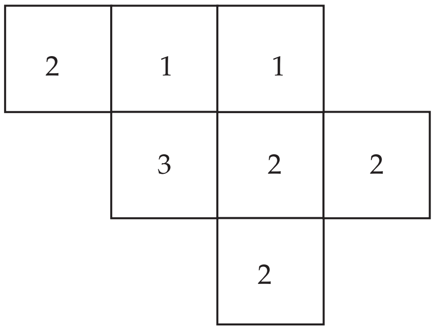
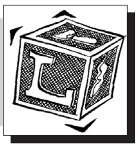
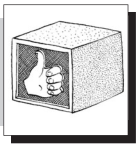




What have you learned?

You can probably see how useful a floor plan can be. When you wrote the numbers in the floor plan it is quick and easy to count how many blocks there are in each building.

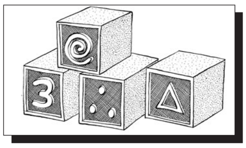


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. I can interpret drawings of the same object from different viewing positions and use the correct terminology to name each view represented.				
2. I can draw and name different views of the same object				
3. I can represent different views of block buildings on a dotted grid.				
4. I can identify the number of blocks used in different block buildings and show this on a numbered floor plan.				
5. I can follow a given floor plan to draw or construct different block buildings.				



Linking your learning with your ECD work

Looking at things from different viewing points

- Let children describe what they see when they look at a familiar object from different viewing positions. A child might say 'When I look at the shoe from up here I can only see the top and the laces'

Building block buildings

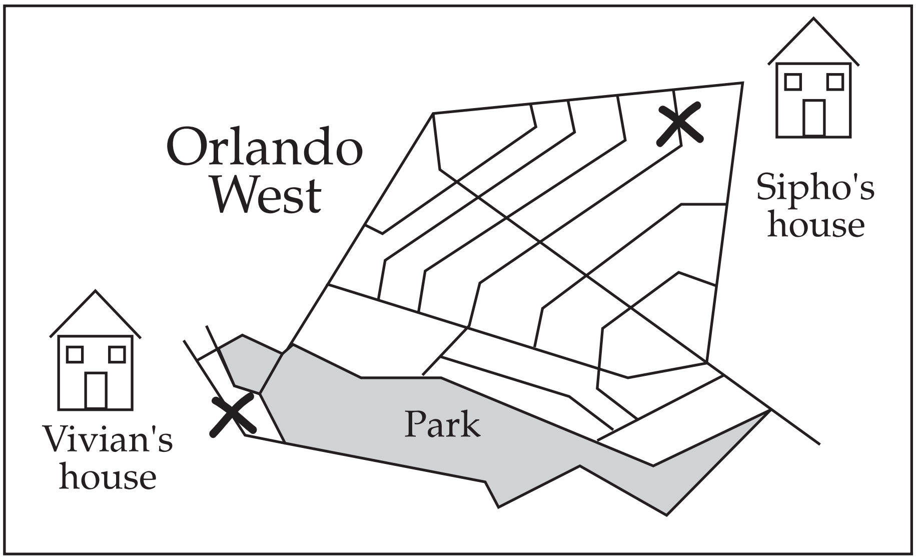
- Give children practice in following your instructions to build block buildings. You might say: 'Put one block in the first row. Now put two layers of two blocks in the row behind that one. Finish with one layer of three blocks behind the second row.'
- Make models using blocks and ask the children to copy them.
- Encourage their use of informal language of position and ordinal numbers as they work, for example, on top of, next to, behind, in front, first second third etc.

Time needed
15 minutes

Activity 6:
Working with maps

Work with a partner

The map below shows you where Vivian and Sipho live in Orlando West.



1. Describe two different routes that you could take from Vivian to Sipho's house.
2. Discuss these questions:
   a. What made it easy and what made it difficult for you to explain these routes?

   b. What other information could have made it easier for you to read the map?
   c. Does this map give you any information about distance from one place to another? How could this be done?

What have you learned?

From this you probably realized that to make things easier, you need the map to have more information so that you can tell exactly where things are and also be able to work out the distance from one place to another.

2. Scale on a map

Maps are drawn to scale because it is impossible to draw the actual size on a sheet of paper. Different maps use different scales. A scale is a ratio where the distance on paper represents a far bigger distance in real life. Let's take the example of the map in Activity 5:

In this map the scale is given as 1:20 000. This means that a distance of 1cm on the map represents 20 000cm in reality. So the actual distance is 20 000 times larger than the distance on the map. 20 000cm = 200m, so this means that 1 cm on the map represents 200m in reality!

If you use a ruler you will find that the distance on the map from Vivian's house to the school is 3cm. This means that the actual distance = 3 × 200m = 600m = 0,6km. Scale can help you to work out approximate distances from one place to another. You can also use this information to work out the approximate time it will take you to get from one place to another.

Activity 7:
Ratio and scales

Time needed
20 minutes

Work Alone

1. Use the scale 1:20 000 and your ruler to calculate the distances of the two routes from Vivian to Sipho's house you found in Question 1.
2. Say which route is shorter.
3. Use the same scale to estimate the border around the park (the perimeter).

Activity 8:
Draw your own map

Time needed
20 minutes

Work alone

1. Draw a sketch that shows a map of your community.
   a. Show on the map the ECD centre where you work or different places that you visit for your work.
   b. Choose four or five other familiar landmarks and show where these are in relation to your place of work.

c. Think about the distance between places and try and show these as realistically as possible on your map.

2. Show your map to a partner and explain how you decided to place the different landmarks in relation to one another. You will come back to this map later after learning about other ways of drawing maps

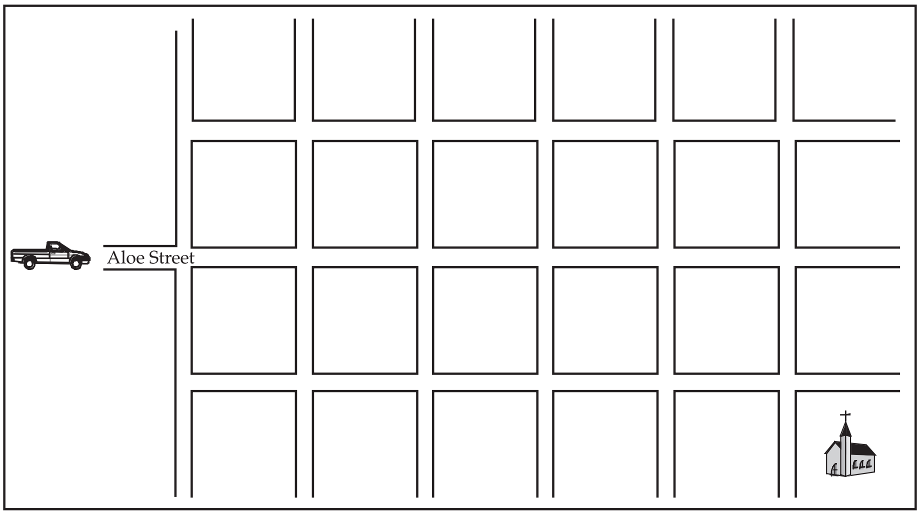
Time needed
15 minutes

Activity 9:
Maps on Grids

Work alone

You are visiting an ECD training agency's offices in Newtown. The secretary gave you these directions:

"When you come into Newtown with Aloe Street, turn left at the first road, then right at the first road, then left at the second road. Our building is the third on the right."



1. Draw the route you will take on this map.
2. Mark with an X where the ECD Agency's offices are.
3. Describe in words a different route to the offices starting from Aloe Street.
4. Give someone directions in words how to get from the ECD Training agency's offices to the church.
5. Give directions in words how to get from the church to the ECD training Agency's offices.
6. Mark a spot anywhere on the map, but do not show it to anyone. Describe to a partner how to get from the church to the spot and show you where this spot on the grid is.

Stop and Think

- What made it easy, what made it difficult to give and to follow directions using this map?
- How could you make it easier to give or follow directions using a map drawn on a square grid like this?
- Where else have you seen examples of maps drawn on square grids before?

3. Coded Grids

In the last activity you read information on a map that was drawn on a grid. You used words like up, down, left, right across to explain how to move from one point to another. The map was not coded. Maybe you found it a little difficult to say exactly where a place was.

In the next map you will notice that the grid is coded. The rows are labelled with letters and the columns are labelled with numbers. This makes it easy to find a place on the grid. You read the row and column references and see where they intersect. For example, you will see on the map on page 122 that Koeberg Station is in row H and column 3. We say that the map co-ordinates for Koeberg are H3.

You will find this grid system of marking the map co-ordinates in a street map.

Activity 10:
Reading maps on a coded grid

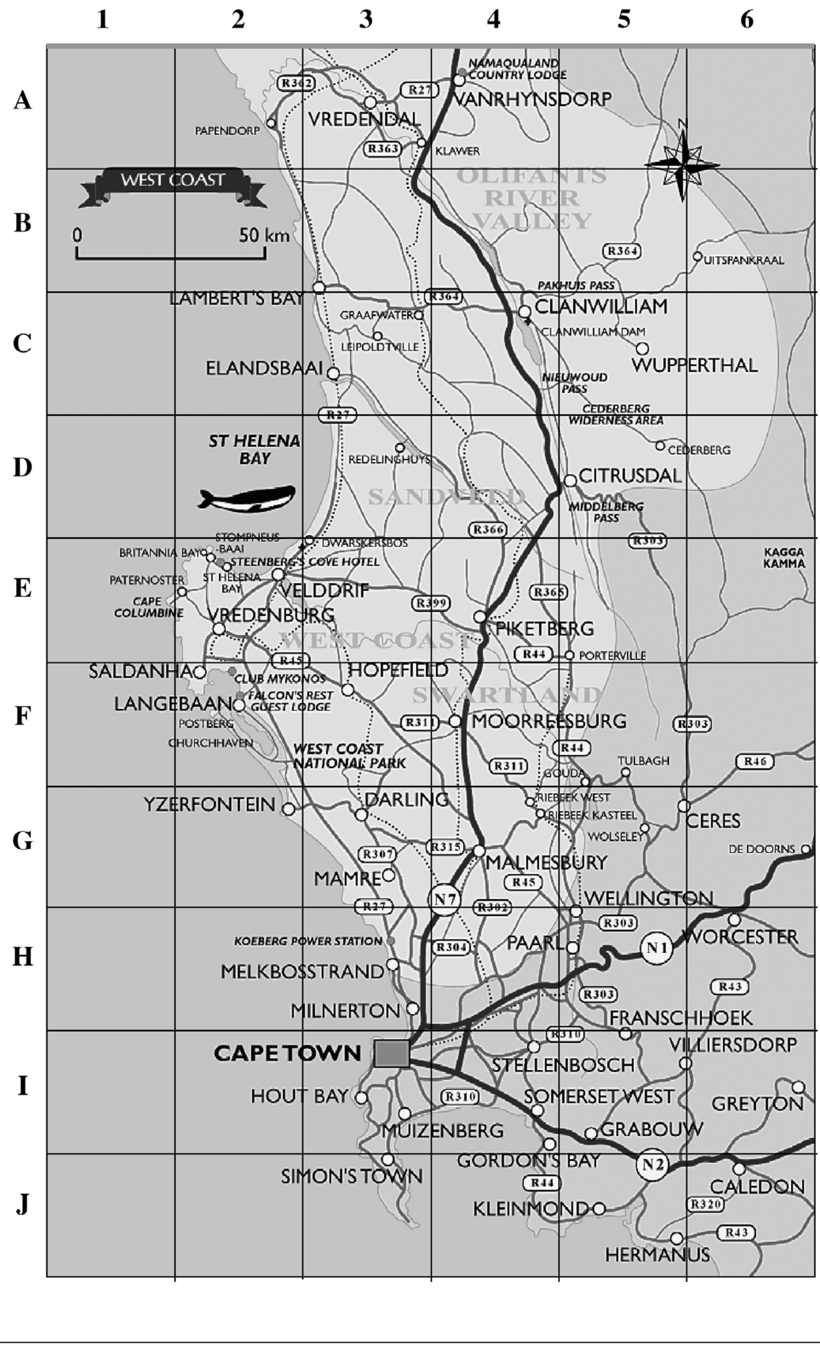
Time needed
30 minutes

Work alone
Refer to the map on the next page for this activity.
Read the information and then answer the questions below:

Mandisa and Sheila are two ECD trainers who visit different ECD sites in the Western Cape. They use a map to help them find their way. The first landmark they pass is the Koeberg Station on the map.

1. List the map co-ordinates for these places that Mandisa and Sheila are going to visit:
   (i) Clanwilliam (ii) Lambertsbaai (iii) Vredendal (iv) Worcester (v) Wuppertal
2. Give the names of three towns in block C3.
3. Which direction is North? How do you know?
4. Name a town to the South of Piketberg (Piketberg is in E4).
5. Name a town to the West of Piketberg.
6. Describe three different routes by road from Cape Town to Velddrif in E2.
   a. Which one do you think is the shortest?
   b. Which one can you drive the quickest?
7. Use the information about scale to make an estimation of the distance by road along the N7 between Cape Town and Van Rhynsdorp. Explain your reasoning in detail.

8. If you drive at an average speed of 120kmph, about how long will it take to drive by car from Cape Town to Van Rhynsdorp? Explain your reasoning in detail.
9. Estimate the area in square kilometres of the land on the map. Explain your reasoning in detail.
10. Estimate the area of the Winelands in square kilometres. Explain your reasoning in detail.
11. Use the scale to find a place that is approximately:
    a) 250km north of Cape Town b) 100km south of Cape Town
12. Share your answers with a partner. Make any corrections that you need to.



Time needed
30 minutes

Activity 11:
Drawing a map on a coded grid

Work alone

In this activity you are going to re-draw the map you drew in Activity 6. You will draw it on a grid showing the map co-ordinates of all the different places you drew in your first sketch.

Keep the sketch of your map in front of you. Follow these steps to guide you:

1. Draw a 10×10 grid on paper with each square measuring 2cm. Label the columns with numbers from 1-10 and the rows with letters from A-J.
2. Estimate the actual distances between the places on your map.
3. Choose an appropriate scale to show the distances between places. For example, 1cm on your map could represent 1km or $\frac{1}{2}$ km in real life. The scale you decide on should work well for the actual distances between the places on your map.
4. On the grid draw an outline of the whole area you want to show.
5. Place the landmarks you chose in the correct places.
6. Write down the map co-ordinates for all of the landmarks you have shown.
7. Share your map with a partner. Let them use the scale you chose to check if you have placed your landmarks correctly and if your scale works.
8. Make any adjustments to your scale that you need to.

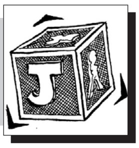
Do this activity on a separate paper and add it to your portfolio

What have you learned?

In these activities you learned about different ways of drawing maps. You first investigated different maps that showed two urban areas where the streets were laid out and you could trace your route from one point to another.

In the last activity you read information from a map that was drawn on a coded grid. This helped you to find exactly where particular places were. The scale of the map helped you to estimate the distance from one place to another.

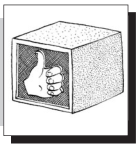
You used what you learned to redraw the map that you sketched earlier. This time you drew it on a coded grid and chose a scale appropriate for the distances you want to show. For example if you show distances in km every cm on your map will represent 1km and not 1m in real life.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about interpreting maps with coded grids?
b. How do you think you will be able to improve your map reading skills?
c. Write down one or two questions that you still have about drawing maps.
d. Write down one or two questions that you still have about choosing a scale.
e. How will you use what you learned about maps in your every day life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Name the different views of the same object, seen from different positions;				
2. Draw different views of the same object seen from different viewing positions;				
3. Draw different views of block "buildings" on a dotted grid;				
4. Use numbers to describe the floor plan for different block "buildings";				
5. Use a given floor plan, represented by numbers, to draw or build different block "buildings"				
6. Interpret maps drawn on coded grids and that use a scale; answer questions relating to the information shown				
7. Draw my own map of a familiar landmarks in your community using the conventions of a coded grid and choosing a scale appropriate to the distances I want to show				